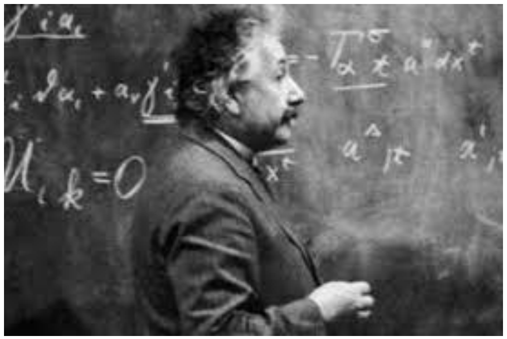
General Relativity (GR)

M1 - Physique 2023-2024



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$$g_{\mu\nu}$$

AE+GR (1907-1917)

$$S[g] = \frac{1}{16\pi G}\int \sqrt{-g}(R-2\lambda)d^4x$$

I) Gravitation before 1916

- Short history of gravitation (Copernicus, Galileo, Kepler, Newton…)
- One unsolved gravitation problem (within the Newton theory) and two tests of GR imagined by A. Einstein
- Predictions of GR (gravitational waves, black holes)

$$R_{ab} - \frac{1}{2}Rg_{ab} + \lambda g_{ab} = 8\pi G T_{ab}$$

$$\ddot{x}^d + \Gamma^d_{ab}\dot{x}^a\dot{x}^b = 0$$

Short history of gravitation

Def (Wikipedia): Gravitation, one of the **four fundamental interactions** governing the Universe, is the physical interaction responsible **for the attraction of massive bodies**. It is manifested in particular by the Earth's gravitational pull, which keeps us grounded and is responsible for many natural phenomena: tides, the orbit of planets around the Sun and the sphericity of most celestial bodies are just a few examples. More generally, **the large-scale structure of the Universe (patterns of galaxies and matter) is determined by gravitation**. Pb of dark matter, dark energy (could be a repulsive force for antimatter, GBAR)…

?

Four: electromagnetic, weak, strong and gravitation

SU(5) grand unification in the standard model

A Grand Unified Theory (GUT) is a theory that unifies the strong, weak, and electromagnetic interactions under a single gauge coupling SU(5)

Def (Wikipedia): In physics, theories of gravitation postulate mechanisms of interaction governing the movements of bodies with mass. There have been numerous theories of gravitation since ancient times…

Short history of gravitation

Main objective for the ancients:

→ to understand the evolution of celestial objects (those we observe)

"The Celestial Atlas, or the Harmony of the Universe‘‘ (from) Andreas Cellarius (1596 – 1665)

Short history of gravitation

Geocentrism

Ptolemy (100-168), Aristote, Tycho Brahe (1546 –1601) , Muslim astronomers…

Heliocentrism

Copernicus (1473-1543)

Galileo (1564-1642)

Brahe (1546-1601), Kepler (1571-1630)

Newton (1642-1727)

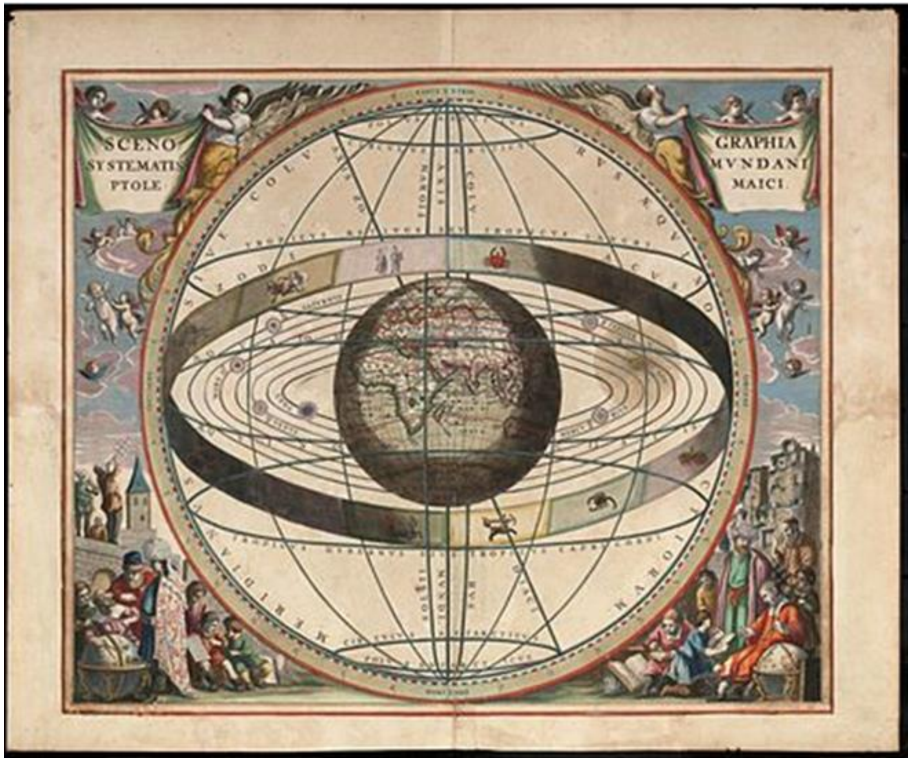
Hilbert (1862-1943)

Einstein (1879-1955)

Short history of gravitation

Ptolemy, Aristote, Tycho Brahe, Muslim astronomers…

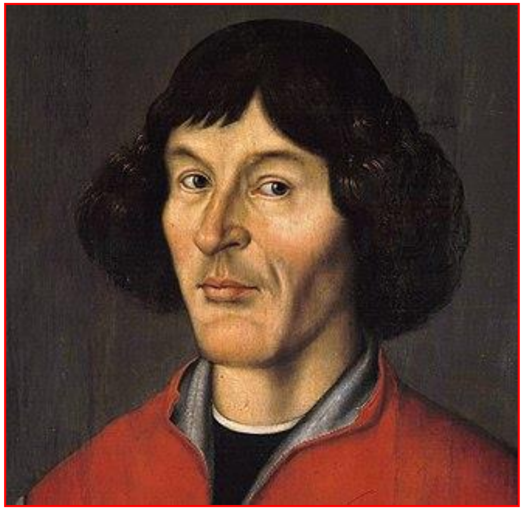
Geocentrism is an ancient physical model which holds that the Earth is immobile and at the center of the Universe. This theory dates back to Antiquity, and was notably defended by Aristotle and Ptolemy. It lasted until the end of the 16th century.



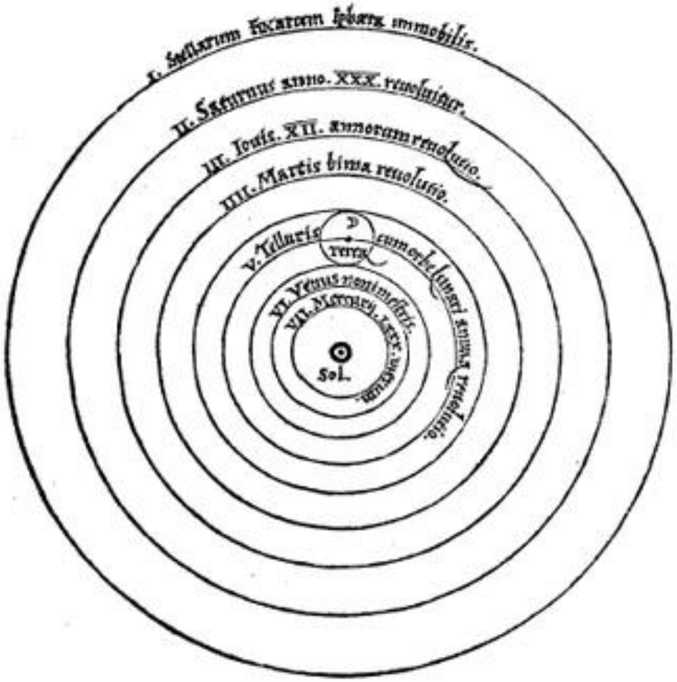
Reproduction of Ptolemy's geocentric system (17th century)

Nicolas Copernic

He is famous for having developed and defended the theory of **heliocentrism**, according to which the Earth revolves around the Sun, assumed to be at the center of the Universe, against the then accepted idea that the Earth was central and immobile. The consequences of this theory, which brought about profound changes in scientific, philosophical and religious opinion, are known as the **Copernican revolution**.



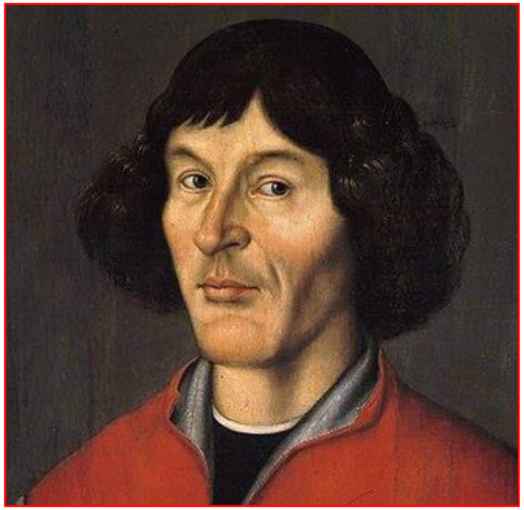
(1473-1543)



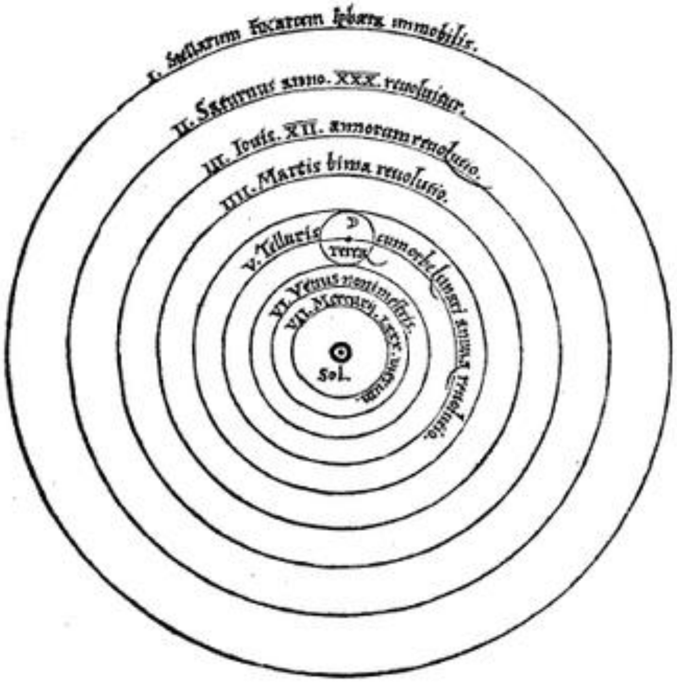
Short history of gravitation

Nicolas Copernic

At the time of Copernicus, there was no need for a paradigm change. However, there were inconsistencies, and Ptolemy's system based on geocentrism was very complex to use.



(1473-1543)



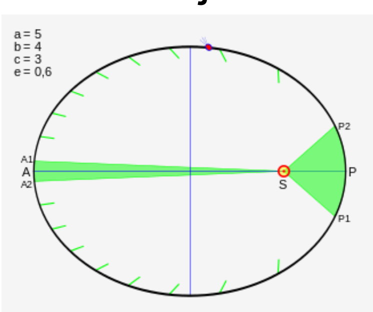
Short history of gravitation

Kepler

(1571-1630)

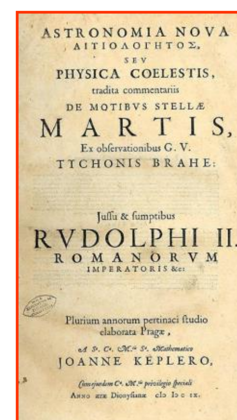
Kepler discovered the three mathematical relationships, known today as Kepler's laws, that govern the motion of the planets in their orbits. The first two were published in 1609 in a work entitled Astronomia Nova. The third was not published until 1618, and quantifies the relationship between the length of the semi-major axis and the period of revolution. These relationships are fundamental, as they were later used by **Isaac Newton** to develop his theory of universal gravitation.

- The planets describe elliptical trajectories with the Sun as their focus.
- Law of areas: the motion of each planet is such that the straight line segment linking the Sun and the planet sweeps equal areas for equal durations.
- Law of periods: the square of a planet's sidereal period T (the time between two successive passages in front of a star) is directly proportional to the cube of the semi-major axis a of the planet's elliptical trajectory, i.e.



Law of areas: each interval corresponds to 5% of the period.

$$\frac{a^3}{T^2} = k$$ with k constant

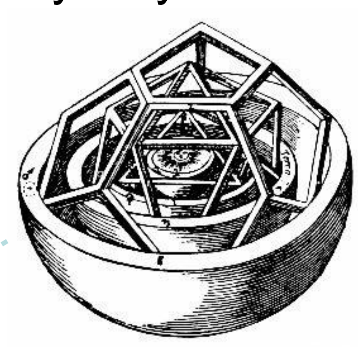


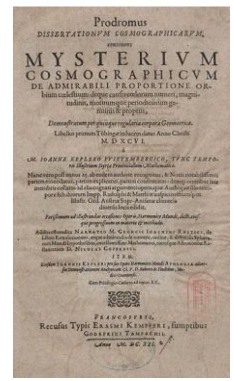
Short history of gravitation

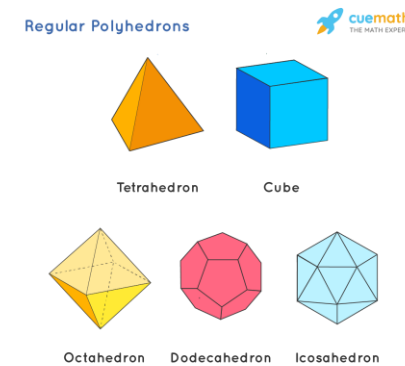
Kepler

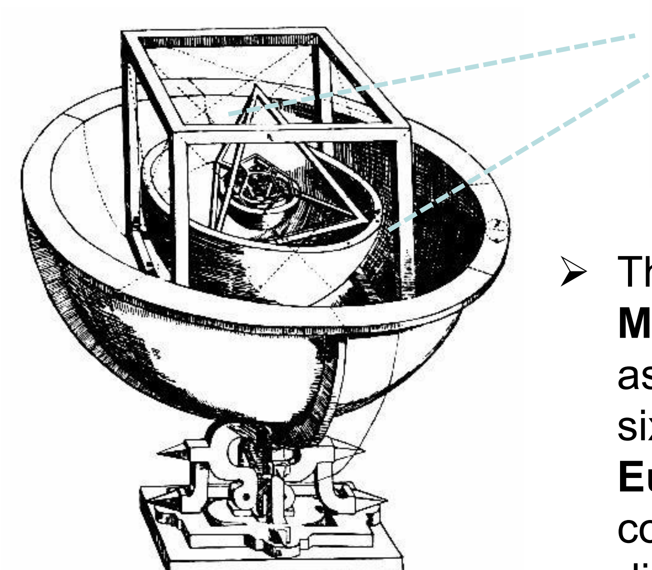
(1571-1630)

- Kepler was an **a priori** Copernican, i.e. he tried to build models compatible with Copernican heliocentrism.
- The first theory, that of regular polyhedra, dates from the Cosmographic Mystery of 1596.









The outer sphere is that of **Saturn**; inside it is the sphere of **Jupiter**.

- The upper image is a closeup of the spheres of inner planets, **Mercury, Venus, Earth, and Mars**. This is a beautiful astronomical model. For example, it explains why there are only six planets: **How could there be a seventh planet, when Euclid proved that there are only five Platonic solids**! Of course, the model is completely false, the interplanetary distances it predicts are not sufficiently accurate, and Kepler was scientist enough to accept this eventually. But it an excellent example of how truth and beauty are not always equivalent.

http://www.georgehart.com/

Short history of gravitation

Kepler (theory) → Tycho Brahe (experiment)

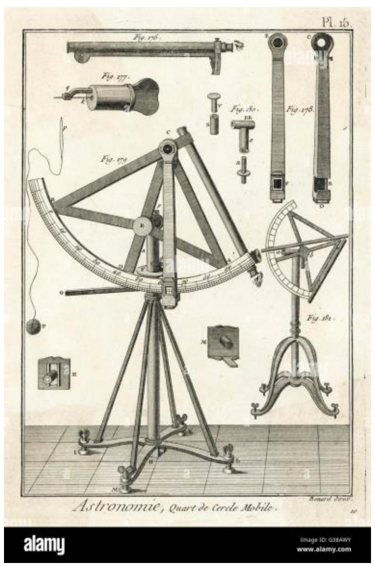
- Tycho Brahe didn't believe in Nicolaus Copernicus' heliocentrism.
- Kepler was invited by Tycho Brahe to Prague in 1600. He remained there until 1612, holding a position as imperial mathematician.
- Brahe (before he died in 1601) asked Kepler to calculate the precise orbit of Mars, whose positions, according to his observations, resisted all attempts at modeling, and deviated significantly (by several degrees) from those predicted by the tables.
- Kepler was born with myopia and diplopia, and therefore was unable to make observations. He based his theories on Brahe's observations.
- Thanks to Tycho Brahe's **meticulous observations (the best and most accurate at the time)**, Johannes Kepler was able to lay the scientific foundations of celestial mechanics based on the heliocentric system.

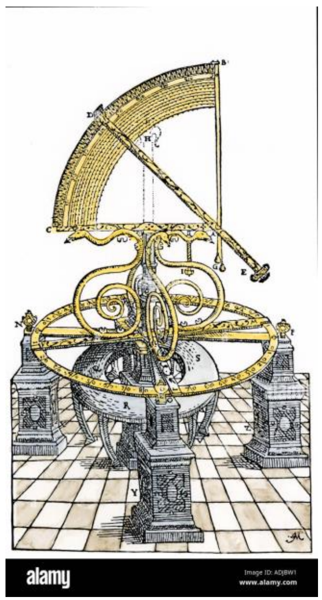
  - https://phys.org/news/2012-11-mercury-poisoning-tycho-brahe-death.html

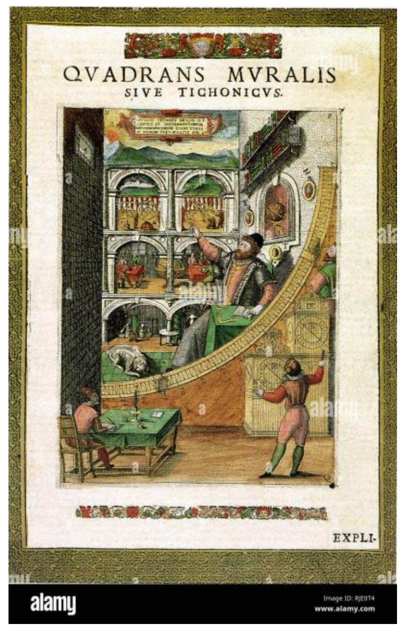
Short history of gravitation

Tycho Brahe (experiment)

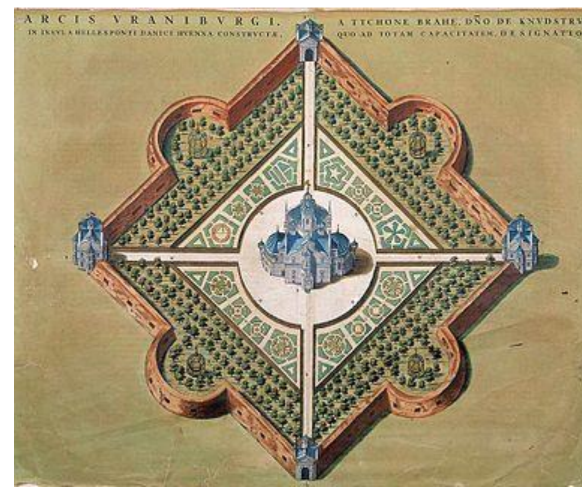
- Tycho Brahe used **astronomical quadrants** (without glass or lenses) to make his measurements.
- He contributed to the development of astronomical instrumentation. For example, he improved the design of quadrants from wood to metal. To achieve this, he collaborated with Swiss watchmakers specializing in precision mechanics.
- He created in 1580 one of the first observatories ever built in Uraniborg, Denmark.







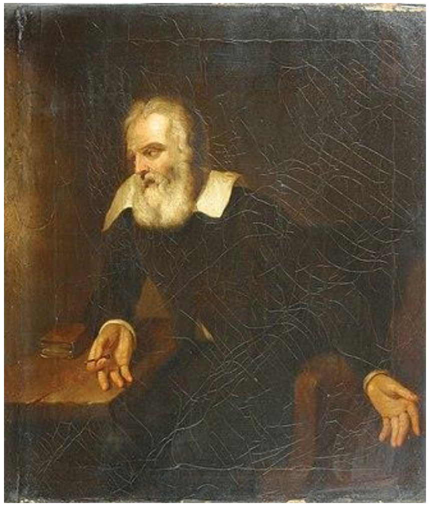
Brahe’s astronomical quadrants



(Wiki) Uraniborg, as presented in Brahe's Astronomiae instauratae Mechanicae in 1598.

Short history of gravitation

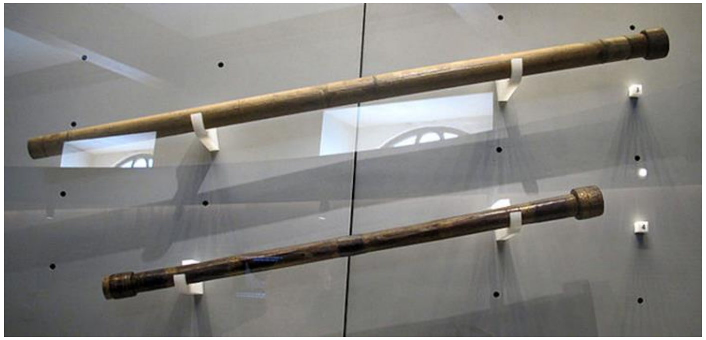
Galileo Galilei



(1564-1642)

In 1609, Galileo learned of the existence of a revolutionary device... Designed by Dutch optician **Hans Lippershey**, the innovation in question was an "instrument for seeing into the distance", which magnified the objects observed by a factor of around **seven**. The Italian mathematician and geometer seized on the principle of the invention and decided to improve it using the elementary principles of optics. He obtained an instrument that magnified linearly up to **thirty times**, and planned to use it to observe... celestial bodies. The telescope became an astronomical telescope, and the physicist an astronomer.

Galileo discovered **Jupiter and its satellites**. He was also the first astronomer to make detailed observations of the **Moon**, **Saturn** and **sunspots** using his own telescope.

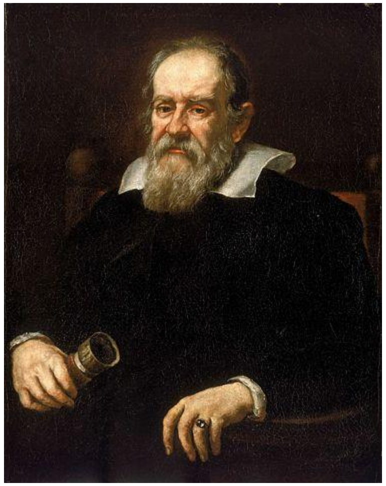


Galileo's telescopes.
Credit: Sailko/CC BY-SA 3.0

https://www.stelvision.com/astro/les-premieres-observations-de-galilee/

Short history of gravitation

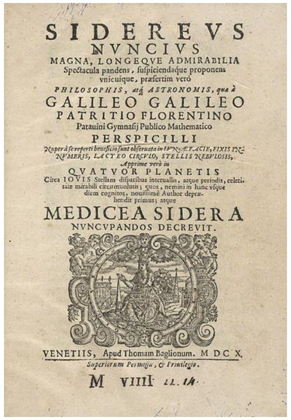
Galileo Galilei



(1564-1642)

Portrait of Galileo (1636).
Credit: Justus Sustermans

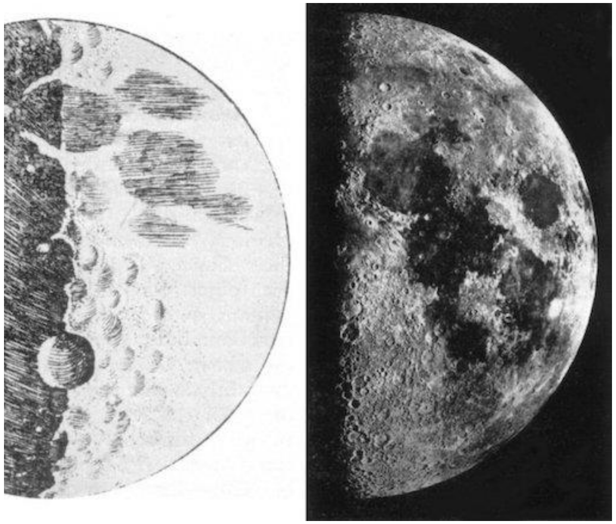
- Galileo used the Copernican tables in his youth.
- **A posteriori**, he looked for observational arguments to confirm Copernicus' model.



SIDEREVS
NVNCIVS
MAGNA, LONGEQVE ADMIRABILIA
Spectacula pandens, suspiciendaque proponens
vnicuique, præsertim verò
PHILOSOPHIS, atq; ASTRONOMIS, quæ à
GALILEO GALILEO
PATRITIO FLORENTINO
Patauini Gymnasij Publico Mathematico
PERSPICILLI
Nuper à se reperti beneficio sunt obseruata in LVNÆ FACIE, FIXIS INNVMERIS, LACTEO CIRCVLO, STELLIS NEBVLOSIS,
Apprime verò in
QVATVOR PLANETIS
Circa IOVIS Stellam disparibus interuallis, atque periodis, celeritate mirabili circumuolutis; quos, nemini in hanc vsque diem cognitos, nouissimè Author depræhendit primus; atque
MEDICEA SIDERA
NVNCVPANDOS DECREVIT.

VENETIIS, Apud Thomam Baglionum. M DC X.
Superiorum Permissu, & Priuilegio.
M VIIII 22.14

Le messager des étoiles
The messenger of the stars



On the left, a drawing of the Moon by Galileo in Sidereus Nuncius, on the right an equivalent photo. https://www.stelvision.com/astro/les-premieres-observations-de-galilee/

Short history of gravitation

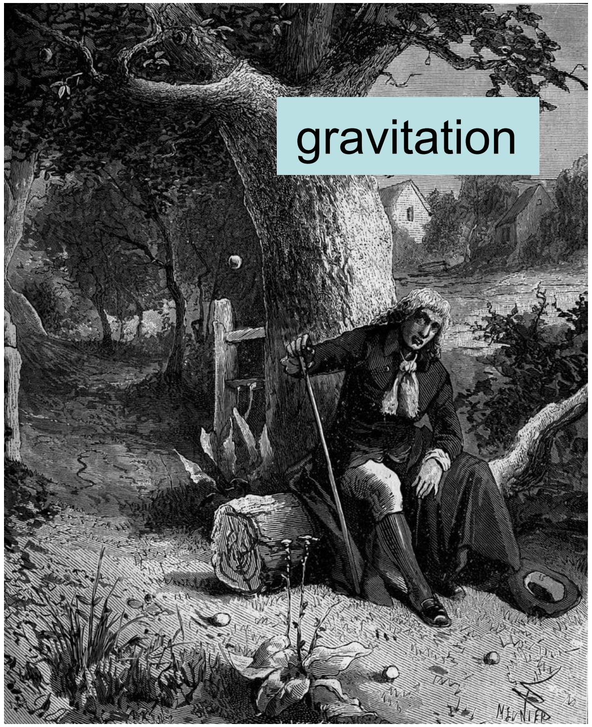
On January 7, 1610, Galileo made one of his most fundamental discoveries with his telescopes. He noticed four small "stars" accompanying the planet Jupiter as it moved...In September 1610, Kepler described these stars as "satellites" in his Narratio, a short and precise account of their observation. **Io**, **Europa**, **Ganymede** and **Callisto** were eventually named by the German astronomer Simon Marius. Galileo was the only person to explain the relative motions of Jupiter and its satellites, which he saw as a reduced model of the solar system. The discovery of these Galilean moons was just one of many proofs provided by the scientist throughout his career, confirming the theory of heliocentrism, according to which the planets revolve around the Sun.

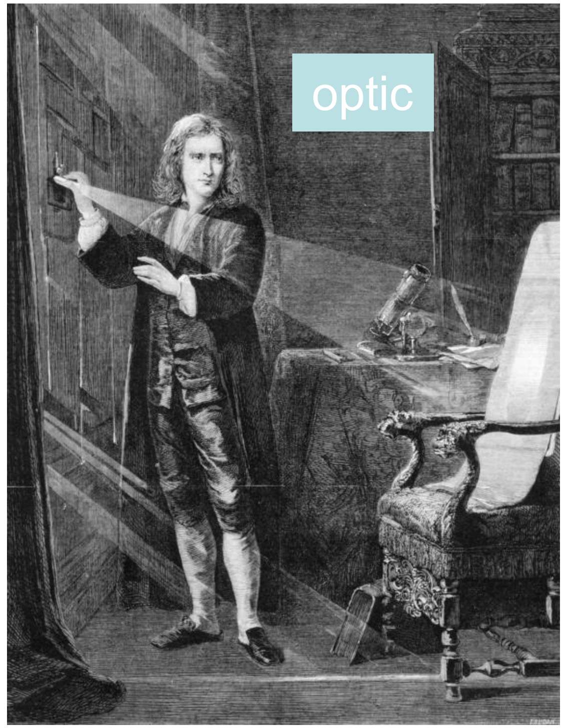
Galileo's defense of Copernican heliocentrism (the Earth revolves daily around the Sun) was opposed by the Catholic Church and certain astronomers. The case was examined by the Roman Inquisition in **1615**, which concluded that heliocentrism was stupid, absurd and heretical, as it contradicted the Holy Scriptures.

Jupiter surrounded by its moons, seen through a telescope at 200x magnification. Credit: Jan Sandberg/Public domain

Short history of gravitation

Isaac Newton (1642-1727)





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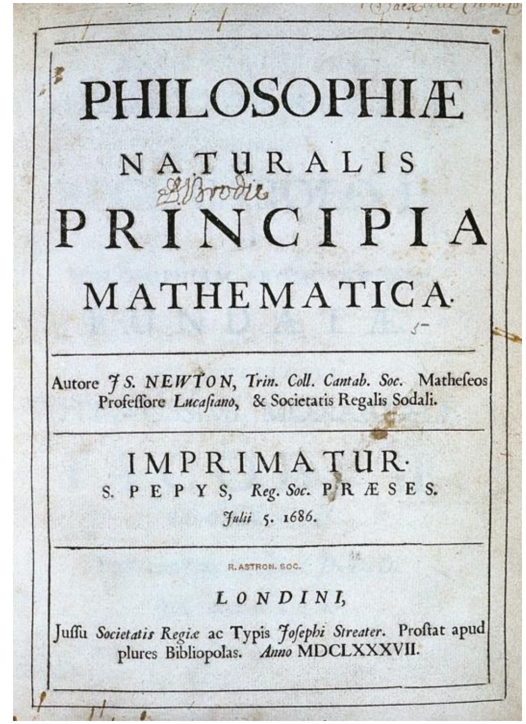

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

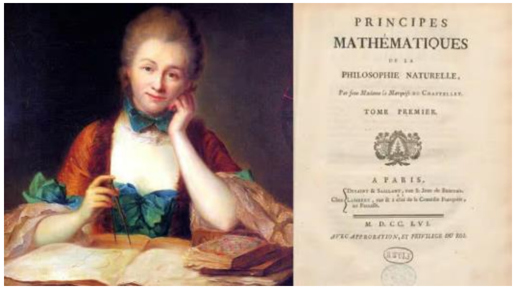
IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

encyclopedia britannica

First edition (1687) in latin!



Principes mathématiques de la philosophie naturelle, translated into French by **Émilie du Châtelet** (1759)

Short history of gravitation

Isaac Newton (1642-1727)

(Wikipedia): Philosophiæ naturalis principia mathematica (Latin for "Mathematical Principles of Natural Philosophy"), often abbreviated to Principia or Principia Mathematica, is Isaac Newton's masterwork. This Latin work was published in London in 1687.

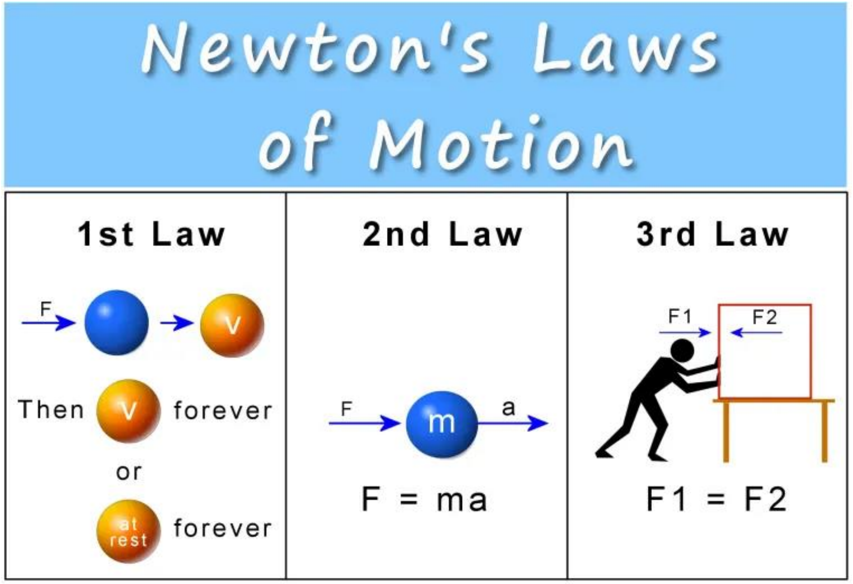
A seminal work

- Philosophiæ naturalis principia mathematica is one of the most important scientific books ever published.
- In this work, Isaac Newton applied "mathematical laws to the study of natural phenomena". The result is what we now call Newton's Laws of Motion, which lay the foundation for **Newtonian mechanics**, as well as **the universal law of gravitation**.
- From these laws, Newton also deduced Kepler's laws of planetary motion, which had been obtained empirically by Johannes Kepler.
- In formulating these physical theories, Newton developed infinitesimal calculus, a field of mathematics. Nevertheless, the language of infinitesimal calculus is largely absent from the Principia, as Newton reformulated most of his demonstrations in geometrical arguments, the common language of physics at the time.

Short history of gravitation

foundation for **Newtonian mechanics**

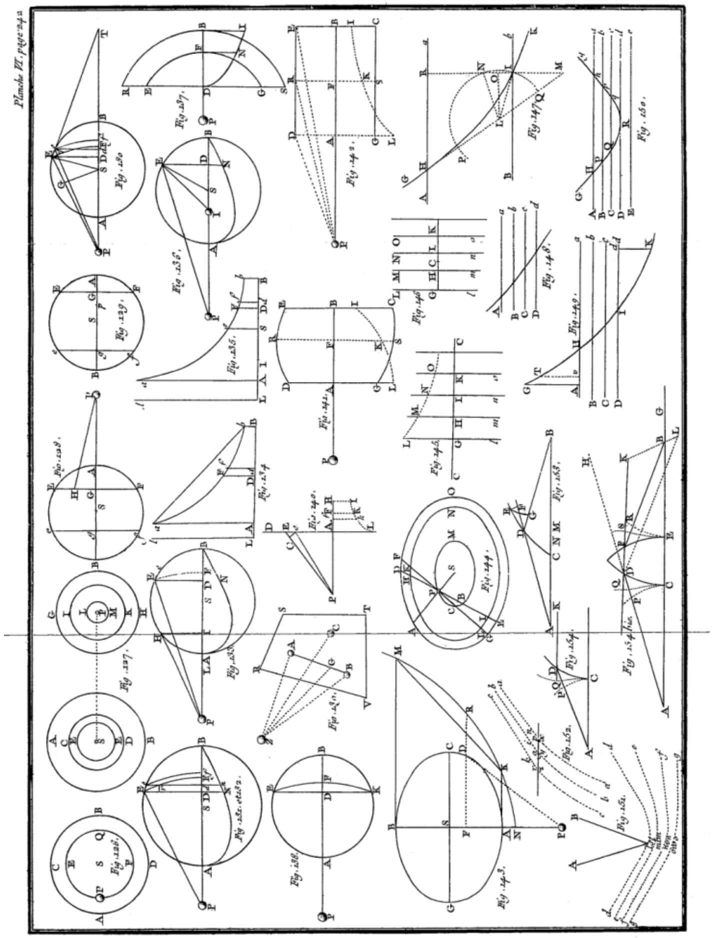
- In the **first** law, an object will not change its motion unless a force acts on it.
- In the **second** law, the force on an object is equal to its mass times its acceleration.
- In the **third** law, when two objects interact, they apply forces to each other of equal magnitude and opposite direction.



Newton's three laws.© Eugene Brennan

Short history of gravitation

The universal law of gravitation



Demonstrations based essentially on geometric considerations

118 PRINCIPES MATHÉMATIQUES

DU MOUVEMENT DES CORPS.

révolution retournent à leurs premieres grandeurs, l'équation reviendra à ſa premiere forme ; ainſi une ſeule & même équation donnera toutes les interſections, & elle aura par conſéquent un nombre infini de racines qui les donneront toutes. On ne peut donc trouver d'une maniere générale une interſection quelconque d'une droite & d'une ſpirale par une équation finie, & par conſéquent il n'y a point d'ovale dont l'aire coupée par des droites à volonté puiſſe être exprimée par une telle équation.

En prenant le rayon de la ſpirale proportionnel au périmetre de l'ovale coupée, il ſera aiſé de prouver par le même raiſonnement qu'on ne peut exprimer la longueur de ce périmetre d'une façon générale par aucune équation finie. Au reſte, je parle ici des ovales qui ne ſont pas touchées par des figures conjuguées qui s'étendent à l'infini.

Cor. De-là on voit que l'aire elliptique décrite autour du foyer ne peut pas être exprimée dans un temps donné par une équation finie, & que par conſéquent elle ne peut être déterminée par la deſcription des courbes géométriquement rationnelles. J'appelle courbes géométriquement rationnelles, celles dont la relation entre les abſciſſes & les ordonnées peut être déterminée par des équations en termes finis. Les autres courbes, telles que les ſpirales, les quadratices, les trochoïdes, &c. je les nomme des courbes géométriquement irrationelles. Je vais montrer à couper l'aire elliptique proportionnellement au temps par une courbe de cette eſpece.

PROPOSITION XXXI. PROBLÉME XXIII.

Trouver pour un temps donné le lieu d'un corps qui ſe meut dans une trajectoire elliptique donnée.

Fig. 79. Que *A* ſoit le ſommet de l'ellipſe *APB*, *S* ſon foyer, *O* ſon centre, & qu'il s'agiſſe de trouver le lieu *P* du corps. Prolongez *OA* en *G*, en ſorte que *OG* : *OA* : : *OA* : *OS* ; élevez la perpendiculaire *GH*, & du centre *O* & de l'intervalle *OG* décrivez

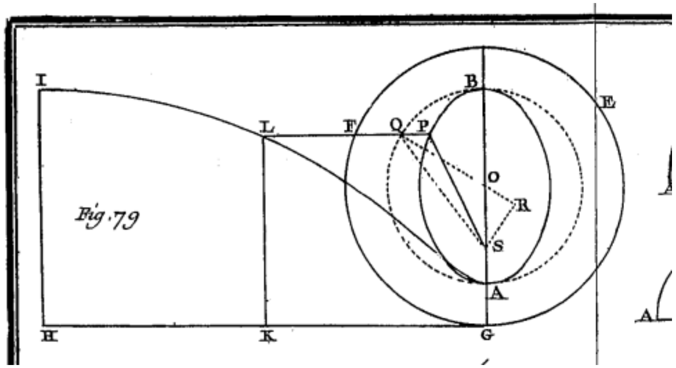
Short history of gravitation

The universal law of gravitation

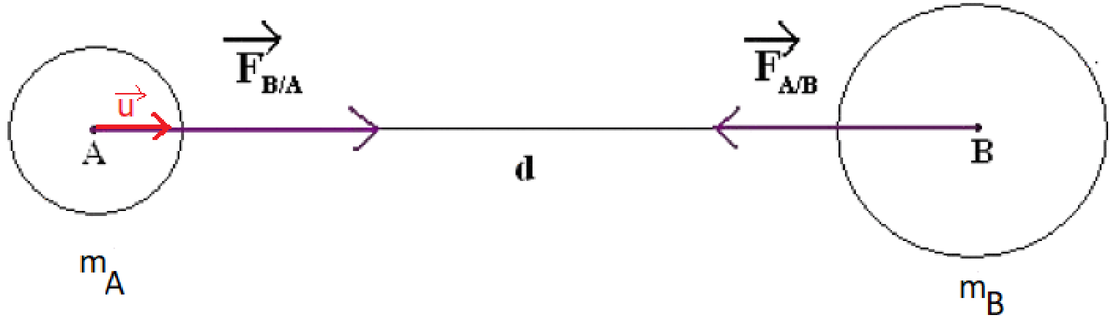
PROPOSITION XXXI. PROBLÊME XXIII.

Trouver pour un temps donné le lieu d'un corps qui se meut dans une trajectoire elliptique donnée.

Fig. 79. Que *A* soit le sommet de l'ellipse *APB*, *S* son foyer, *O* son centre, & qu'il s'agisse de trouver le lieu *P* du corps. Prolongez *OA* en *G*, en sorte que *OG* : *OA* :: *OA* : *OS*; élevez la perpendiculaire *GH*, & du centre *O* & de l'intervalle *OG* décrivez



- Newton rediscovered Kepler's laws, using an attractive force between two massive bodies in inverse proportion to the square of the distance between them.

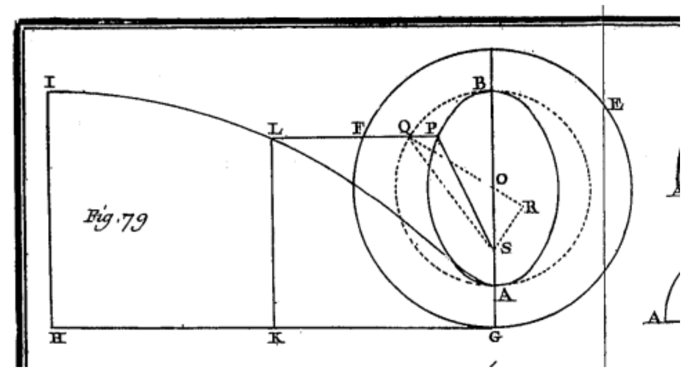


- First use of the word **gravitation** in the Philosophiæ naturalis principia mathematica

Short history of gravitation

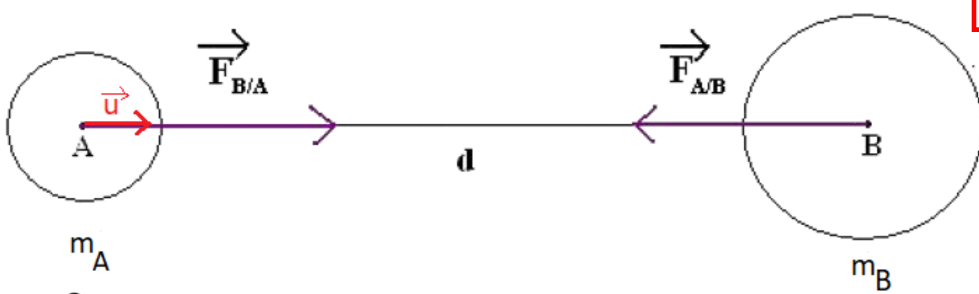
The universal law of gravitation

> PROPOSITION XXXI. PROBLÊME XXIII.
>
> *Trouver pour un temps donné le lieu d'un corps qui se meut dans une trajectoire elliptique donnée.*
>
> Fig. 79. Que *A* soit le sommet de l'ellipse *APB*, *S* son foyer, *O* son centre, & qu'il s'agisse de trouver le lieu *P* du corps. Prolongez *OA* en *G*, en sorte que *OG* : *OA* : : *OA* : *OS*; élevez la perpendiculaire *GH*, & du centre *O* & de l'intervalle *OG* décrivez



Newton rediscovered Kepler's laws, using an attractive force between two massive bodies in inverse proportion to the square of the distance between them.

$$\vec{F}_{\mathrm{B/A}} = G\frac{m_{\mathrm{A}} m_{\mathrm{B}}}{d^2}\vec{u}$$



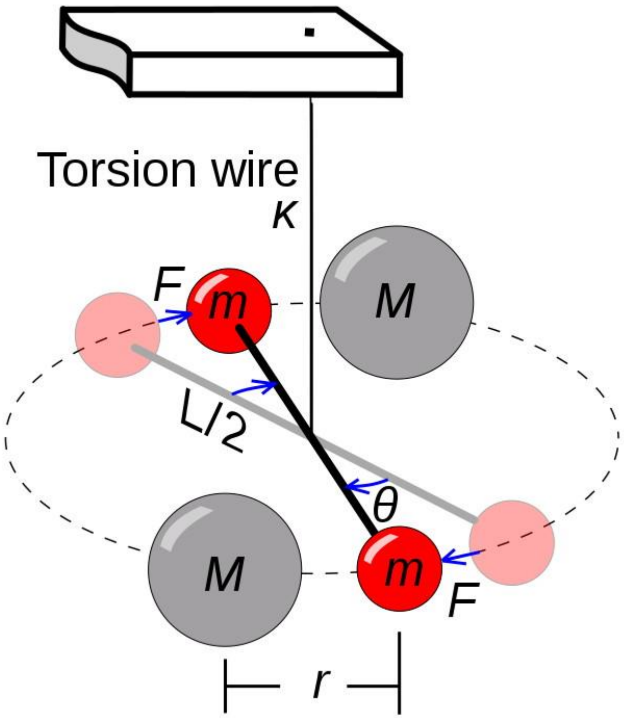
$$||\vec{u}||^2 = 1$$

G: Universal Gravitational constant = 6.67408×10^{-11} Nm^2/kg^2

Short history of gravitation

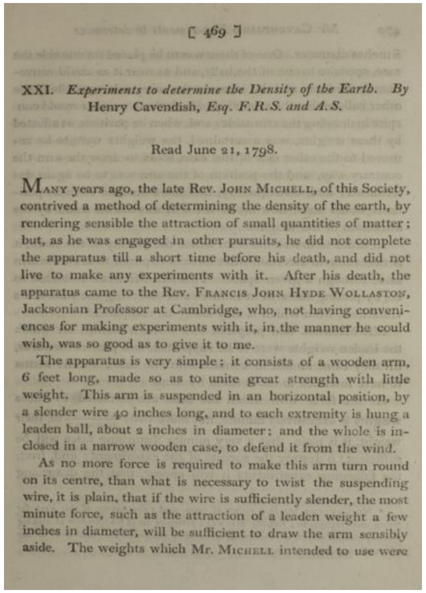
Cavendish's measurements resulted in an experimentally determined value of 6.75 x 10^{-11} Nm^2/kg^2. Today, the currently accepted value is 6.67408 x 10^{-11} Nm^2/kg^2. The value of ***G*** is an extremely small numerical value.

Cavendish, Henry (1798). "Experiments to Determine the Density of the Earth"
Philosophical Transactions of the Royal Society. ***88****: 469–526.*



The **Cavendish experiment**, performed in 1797–1798 by English scientist **Henry Cavendish**, was the first experiment to measure the force of gravity between masses in the laboratory and the first to yield accurate values for the gravitational constant.

- m = 0.73 kg (lead)
- M = 158 kg (lead)
- $F = 1.74\times10^{-7}$ N


[469]

XXI. *Experiments to determine the Density of the Earth.* By Henry Cavendish, *Esq. F.R.S. and A.S.*

Read June 21, 1798.

Many years ago, the late Rev. John Michell, of this Society, contrived a method of determining the density of the earth, by rendering sensible the attraction of small quantities of matter; but, as he was engaged in other pursuits, he did not complete the apparatus till a short time before his death, and did not live to make any experiments with it. After his death, the apparatus came to the Rev. Francis John Hyde Wollaston, Jacksonian Professor at Cambridge, who, not having conveniences for making experiments with it, in the manner he could wish, was so good as to give it to me.

The apparatus is very simple; it consists of a wooden arm, 6 feet long, made so as to unite great strength with little weight. This arm is suspended in an horizontal position, by a slender wire 40 inches long, and to each extremity is hung a leaden ball, about 2 inches in diameter; and the whole is inclosed in a narrow wooden case, to defend it from the wind.

As no more force is required to make this arm turn round on its centre, than what is necessary to twist the suspending wire, it is plain, that if the wire is sufficiently slender, the most minute force, such as the attraction of a leaden weight a few inches in diameter, will be sufficient to draw the arm sensibly aside. The weights which Mr. Michell intended to use were

https://en.wikipedia.org/wiki/Cavendish_experiment

By Chris Burks (Chetvorno) - Own work
This vector image was created with Inkscape.,
Public Domain, Link

Short history of gravitation

Clifford M. Will, page 11 **Principle of Equivalence (Newton)**

- Newton regarded this principle as such a **cornerstone** of mechanics.
- The principle of equivalence demanded that the « mass » of any body, namely that property of a body (inertia) that regulates its response to an applied force, be equal to its « weight », that property that regulates its response to graviation.

Newton's second law → $\vec{F} = m_I \vec{a}$

Law of gravitation → $\vec{F} = m_P \vec{g}$

m_I « Inertial mass » (Bondi 1957)

m_P « Passive gravitational mass »

Principle of Equivalence → $\boxed{m_P = m_I}$

- Alternative statement of this principle is that all bodies fall in a gravitational field with the same accelaration regardless of their mass or internal structure.
- Newton's equivalence principle = « Weak Equivalence Principe » (**WEP**)

https://fr.wikipedia.org/wiki/Fichier:Apollo_15_feather_and_hammer_drop.ogv

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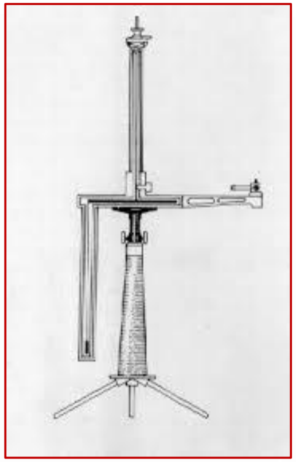
https://fr.wikipedia.org/wiki/Fichier:Apollo_15_feather_and_hammer_drop.ogv

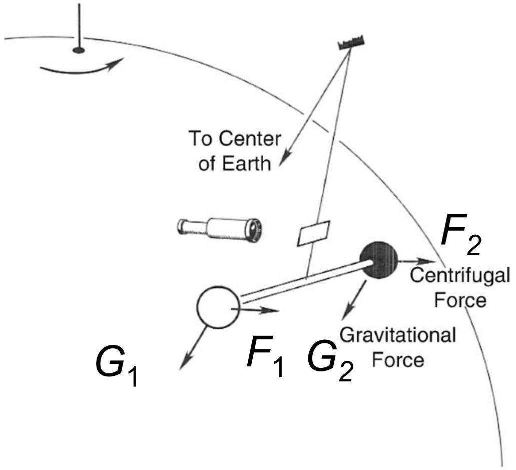
- After EEP: « Einstein Equivalence Principle »

Short history of gravitation

Clifford M. Will, page 18

Principle of Equivalence (Eötvös/Exp 1922)





If the ratio of F_1 to F_2 differed from the ratio of G_1 to G_2, the rod would rotate. The mirror is used to monitor this rotation.

Eötvös's original experimental device consisted of two masses on opposite ends of a rod, hung from a thin fiber. A mirror attached to the rod, or fiber, reflected light into a small telescope. Even tiny changes in the rotation of the rod would cause the light beam to be deflected, which would in turn cause a noticeable change when magnified by the telescope.

As seen from the Earth's frame of reference (or "lab frame", which is not an inertial frame of reference), the primary forces acting on the balanced masses are the string tension, gravity, and the **centrifugal force due to the rotation of the Earth**. Gravity is calculated by Newton's law of universal gravitation, which depends on gravitational mass. The centrifugal force is calculated by Newton's laws of motion and depends on inertial mass.

One unsolved gravitation problem

URANUS

Jean Joseph Le VERRIER URBAIN (1811-1877)

• French astronomer, specialist in celestial mechanics

• The observed orbit of Uranus showed unexplained discrepancies with the calculated orbit.

•Le Verrier tackled the problem in the **early summer of 1845**.

• **On September 18, 1846**, Le Verrier sent his calculations to Galle, Director of the Berlin Observatory…

• **On September 23, 1846**, the very day he received a letter from Le Verrier specifying the object's position, the Berlin astronomer observed the predicted star in the area of the sky indicated by Le Verrier.

• In front of the Académie des Sciences, Arago declared:

«M. Le Verrier a vu un astre au bout de sa plume !»

M. Le Verrier saw a star at the end of his pen!

• The new planet has been named **Neptune**.

- http://www.bibnum.education.fr/physique/astronomie/la-decouverte-de-neptune-1846

The three classical tests of GR

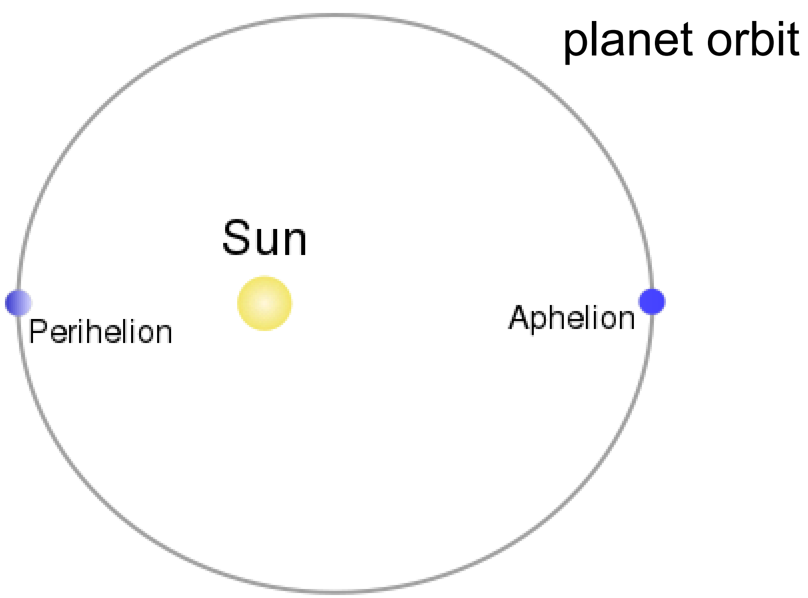
Three classic tests that make the difference between the new theory of gravitation and Newton's: **an anomaly it doesn't explain, and two new effects imagined by A. Einstein**.

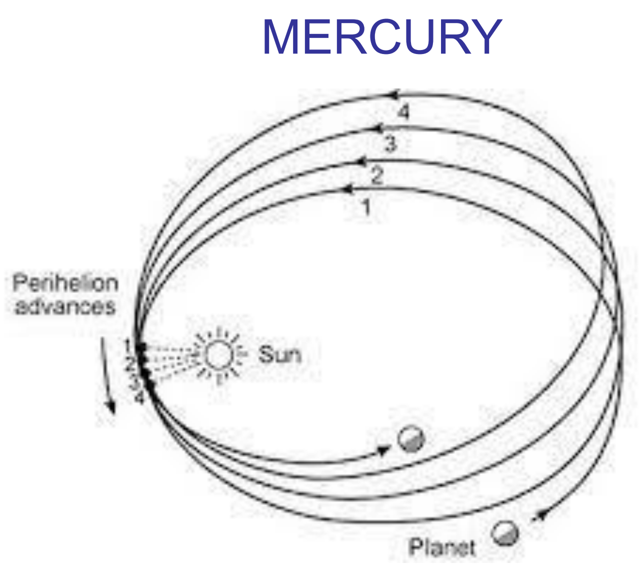
- Perihelion of the planets, in particular Mercury, to which Newton's theory has been unable to provide a satisfactory answer since the middle of the 19th century.

- Prediction of the effect of gravitation on the propagation of light rays. If a light ray passes close to a massive body, it will be subjected to its gravitational field and will be deflected. First verified by **Arthur Eddington in 1919**. Several since, the latest very recently.

- The Einstein effect. This is a test of one of the theory's principles: the equivalence principle. It involves measuring the influence of gravitation on the operation of clocks located in different places and subjected to different gravitational fields. These clocks are present everywhere in the universe: they are the spectral lines of atoms. These lines should be red-shifted in a gravitational field. Many observations and experiments were needed before we were really convinced of its validity in 1960. We then succeeded in setting up a terrestrial experiment along these lines. **This was the first GR experiment**. This effect must be taken into account for accurate GPS measurements (see next page).

One unsolved gravitation problem

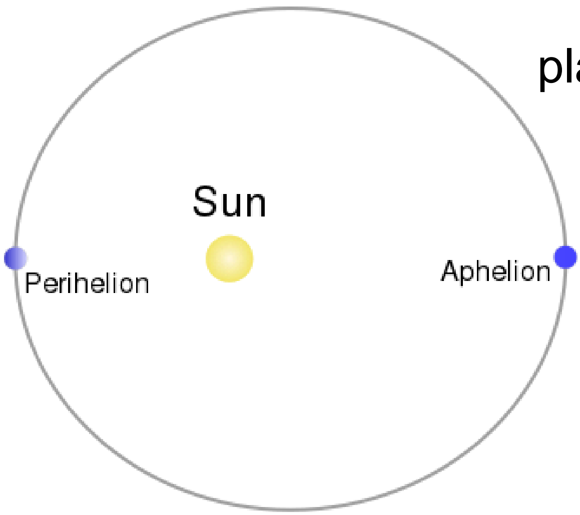




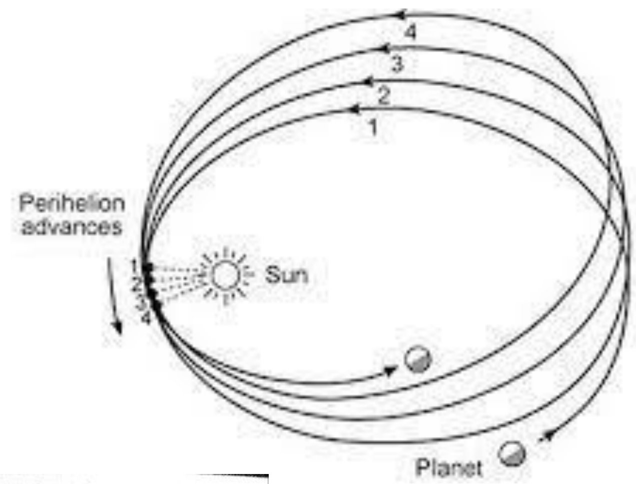
- Mercury undergoes a <u>slight perturbation</u> in its motion compared with the model predicted by Newtonian mechanics, called perihelion precession. With each revolution around the Sun, the perihelion of Mercury's orbit advances slightly as it rotates around the Sun. All planets experience this phenomenon, <u>which is due to the gravitational influence of other bodies</u> in the solar system and can be explained by the classical laws of celestial mechanics. For Mercury, on the other hand, there remains a very slight advance of **43 arcseconds** per century compared with the orbit calculated using Newton's and Kepler's laws.

One unsolved gravitation problem

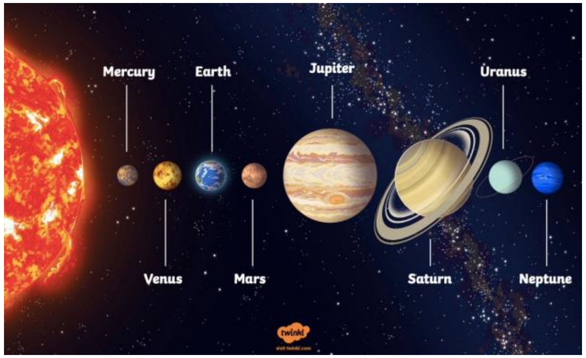
planet orbit



MERCURY

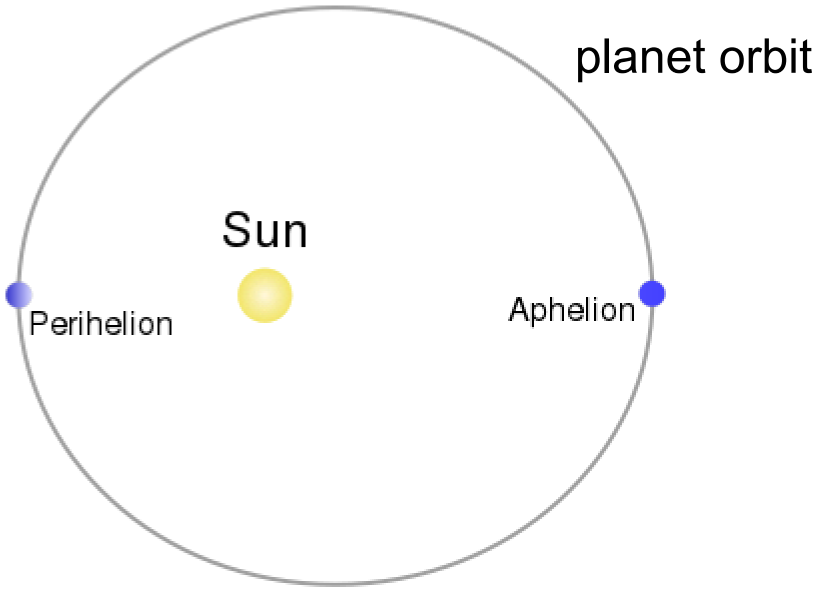


Planet	Advance
Venus	277.8
Earth	90.0
Mars	2.5
Jupiter	153.6
Saturn	7.3
Total	531.2
Observed	574.1
Discrepancy	42.9

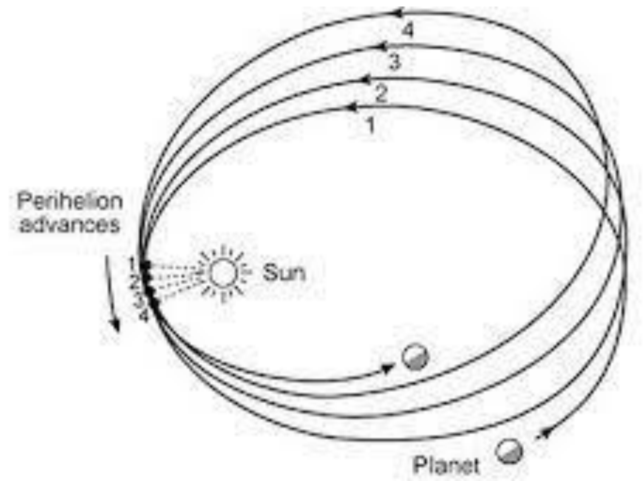


Planetary contributions to Mercury's perihelion advance (in arcseconds per century)

One unsolved gravitation problem



MERCURY



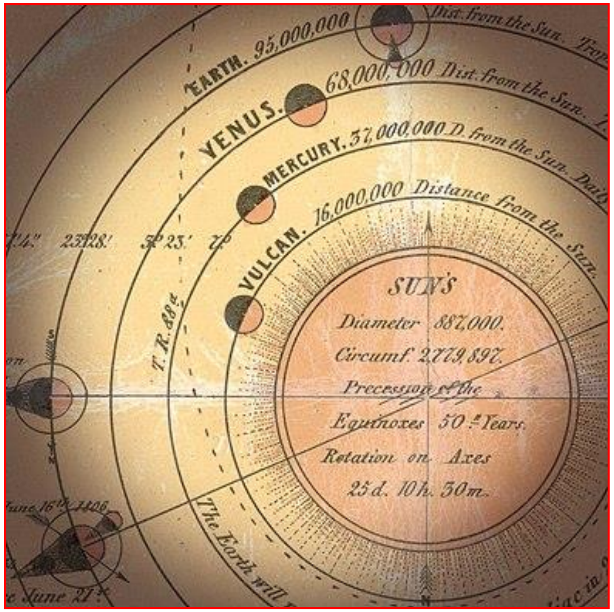
Video:

https://www.google.com/search?q=perihelion+of+Mercury%27s+orbit&source=lmns&tbm=vid&bih=491&biw=1056&client=firefox-b-d&hl=fr&sa=X&ved=2ahUKEwiVmtKCoumAAxXOmycCHSXKAXAQ0pQJKAJ6BAgBEAY#fpstate=ive&vld=cid:073a6be9,vid:NXlg3nTqSnk

One unsolved gravitation problem

MERCURY

- To solve the problem of the advance of the perihelion of Mercury's orbit, Le Verrier postulated the presence of one (or more) bodies between the Sun and Mercury's orbit, whose gravitational interaction would disrupt Mercury's motion, like that of Uranus. He called this celestial body Vulcan.



- The only irreducible discrepancy Le Verrier found in the comparison between observed and calculated orbits was a residual of 38" for the secular motion of **Mercury's perihelion**. It was precisely this residual, confirmed and only slightly modified, that would provide the first material proof of the theory of **general relativity** half a century later.

One unsolved gravitation problem

$$\Delta\varphi_{\text{Einstein}} = \frac{24\pi^3 a^2}{T^2 c^2 (1 - e^2)}$$

LL98,7

Mercury

$$\begin{cases} a \approx 5.8 \times 10^{10} \text{ m} \\ e \approx 0.2 \\ T \approx 88 \text{ days} \end{cases}$$

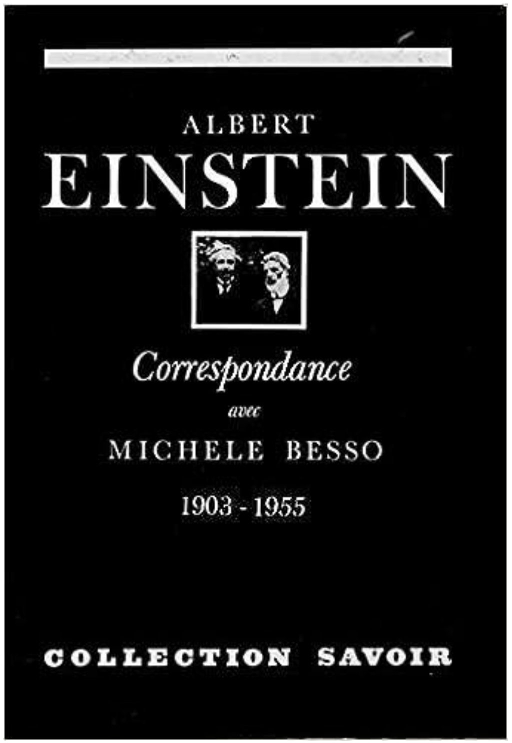
$$\Delta\varphi_{\text{Einstein}} \approx 0.1038 \text{ arcsecond/period}$$

$$\Delta\varphi_{\text{Einstein}} \approx 0.1038 \times 415 \approx 43.03 \text{ arcseconds/century}$$

$$1 \text{ arcsecond} = 1'' = 1^\circ/3600$$

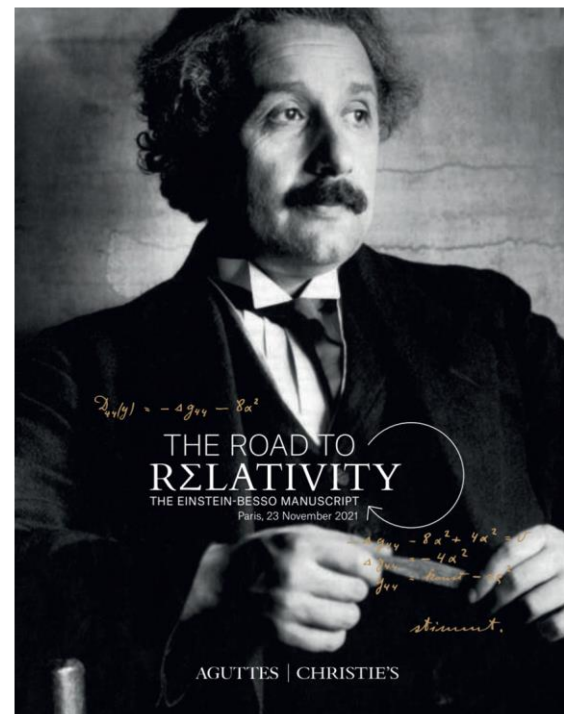
Albert Einstein and Michele Besso

Michele Besso (1873-1955) was a Swiss physicist of Italian Jewish origin, known as Albert Einstein's friend and confidant during his most emblematic work on the theories of special relativity and general relativity.



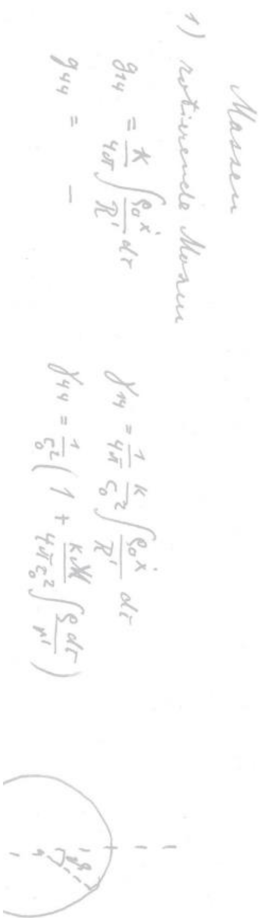
- The best-known aspect of Michele Besso's influence on Einstein's work is a piece of work carried out in late 1913-early 1914, when the definitive version of the equations of general relativity had not yet been written.

- 11.6 million euros for Albert Einstein's most valuable scientific manuscript.



https://www.passeisme.com/articles/116-millions-deuros-pour-le-plus-precieux-manuscrit-scientifique-dalbert-einstein/

Albert Einstein / precession of the…



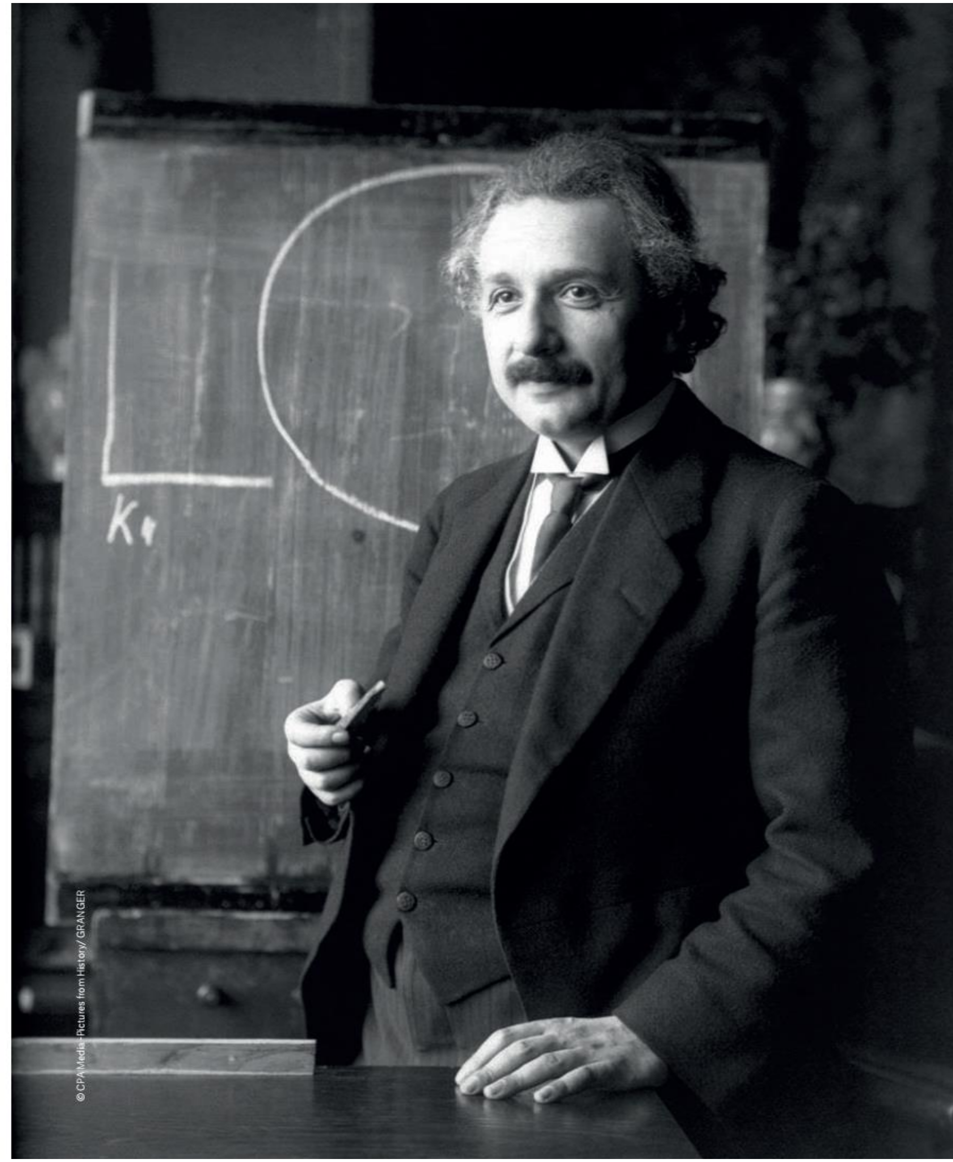
THE SEARCH FOR PROOF OF EINSTEIN'S GENERAL THEORY OF RELATIVITY:
THE EINSTEIN-BESSO CALCULATIONS OF THE PRECESSION OF THE PERIHELION OF MERCURY.

ONE OF ONLY TWO SURVIVING WORKING SCIENTIFIC MANUSCRIPTS BY EINSTEIN FROM THIS KEY PERIOD, AND THE MOST VALUABLE EINSTEIN MANUSCRIPT TO BE OFFERED AT AUCTION.

A CRUCIAL STAGE IN THE DEVELOPMENT OF THE GENERAL THEORY OF RELATIVITY, WHICH RESHAPED MODERN UNDERSTANDING OF HOW THE UNIVERSE WORKS.

The Einstein-Besso manuscript documents a crucial stage in the development of the general theory of relativity, often described as the most beautiful theory in physics, and one of the single most important ideas in modern science. It is a 54-page working manuscript written jointly by Albert Einstein and Michele Besso between June 1913 and early 1914.

Einstein's scientific autographs from this period, and more generally from before 1919, are rare. As one of only two surviving manuscripts documenting the genesis of general relativity, it provides a remarkable insight into Einstein's work and a fascinating glimpse into the mind of the greatest scientist of the 20th century.



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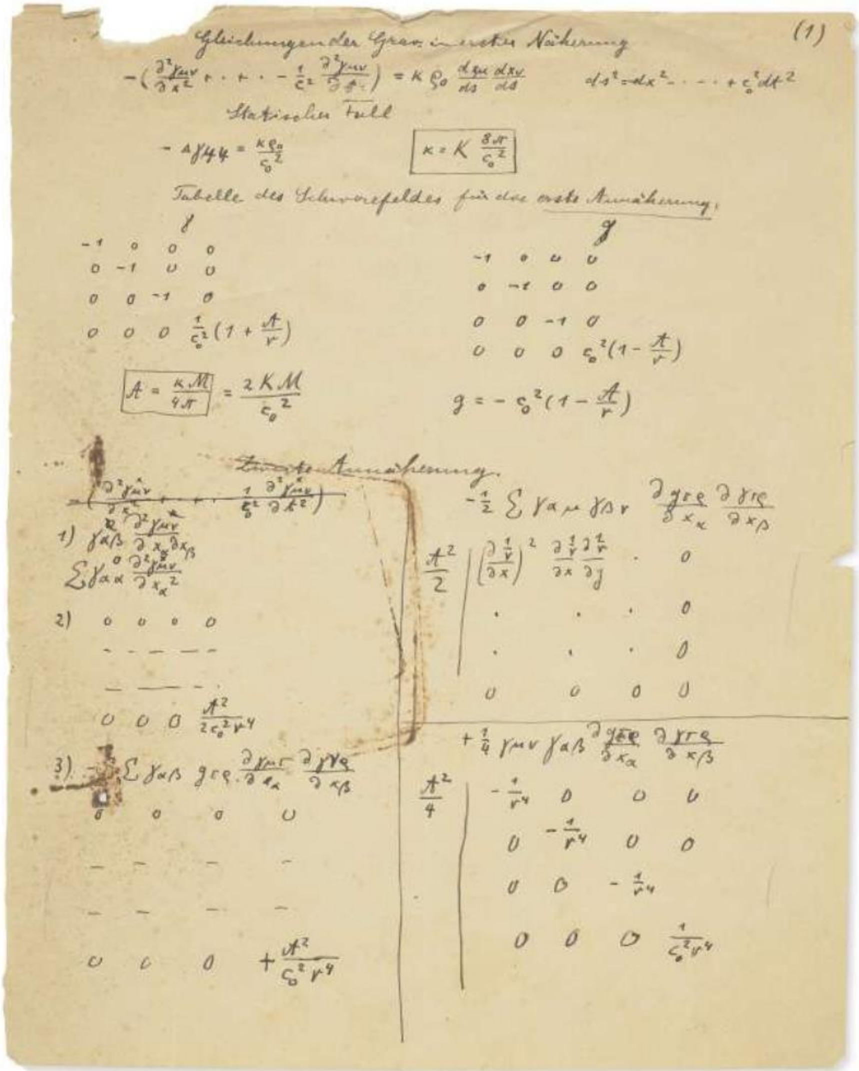
Albert Einstein / precession of the…



ALBERT EINSTΣIN (1879-1955) + MICHELE BΣSSO (1873-1955)

[FR]

Manuscrit autographe avec une série de calculs, utilisant la première version (« Entwurf ») des équations de la théorie de la relativité générale d'Einstein, dans le but de vérifier si la théorie permettait d'expliquer l'anomalie du mouvement du périhélie de Mercure. Sans lieu ni date [Zurich, circa juin 1913 pour la majorité du manuscrit– quelques ajouts début 1914].

26 pages de la main d'Einstein, 25 de Besso, et 3 pages avec des interventions des deux scientifiques (de nombreuses pages présentent des ajouts et corrections de l'un sur des écrits de l'autre) : au total 54 pages de format in-4 (sur 56) à l'encre (quelques passages de Besso au crayon), sur 37 feuillets, pour la plupart 273 x 212 mm (10¾ x 8 3/8 in.), le feuillet [10-11] de format oblong, plié en deux, avec au verso des fragments de diagrammes et calculs (compté dans le total ci-dessus), pagination irrégulière et incomplète (ordonné suivant l'ordre de publication dans les *Collected Papers*).
Condition : un feuillet sans sa partie inférieure (probablement ôtée par l'un des deux scientifiques), page [3] rédigée au verso d'un feuillet imprimé (daté « Ende April 1913 »), une large portion des pages avec importantes corrections aux formules ou passages entièrement barrés, quelques défauts marginaux, sans les pp. [16-17] (ces pages, de la main de Besso, se trouvent au verso d'une lettre adressée par C.-E. Guye à Einstein le 31 mai 1913, aujourd'hui dans une collection privée), p. [1] avec traces de rouille probablement dues à une attache métallique, sinon en bonne condition s'agissant d'un manuscrit de travail.
Publié dans : Einstein, *Collected Papers*, 4:344-359 (introduction), 4:360-473 (transcription, avec notes), and 4:630-682 (facsimilé).

Emboîtage sur mesure d'Alain Taral en ébène de Macassar, chaque feuillet dans son intercalaire individuel, numéroté et avec encadrement rigide pour faciliter sa manipulation.

Provenance: Christie's New York, 25 novembre 1996, – Harvey Plotnick ; sa vente, Christie's New York, 4 octobre 2002, lot n°81 – acquis lors de cette vente par le propriétaire actuel.

€2,000,000-3,000,000

US$2,400,000-3,500,000
£1,900,000-2,700,000

[EN]

Autograph scientific manuscript, a series of calculations using the early version ('Entwurf') of the field equations of Einstein's general theory of relativity, intended to test whether the theory could account for the anomaly in the precession of the perihelion of Mercury. No place, [Zurich], n.d. [mostly June 1913; additions from early 1914].

26 pages in Einstein's hand and 25 pages in Besso's, with 3 pages with entries of both collaborators (many pages with contributions of one to entries of the other): altogether 54 (of 56) pages, written in ink (a few portions by Besso in pencil), on 37 separate leaves of foolscap and squared paper of various types, mostly 273 x 212 mm (10¾ x 8 3/8 in), leaf [10-11] an oblong folded sheet bearing unpublished partial diagrams and calculations on its verso (counted in above total), irregular partial pagination (now arranged as published in *Collected Papers*).
Condition: one sheet with lower portion torn away (probably by the collaborators), page [3] written on the back of a printed announcement (dated 'Ende April 1913'), many pages with extensive corrections to the formulae or with whole sections of calculations crossed out, a few pages with minor corner defects, pp.[16-17] not present (these pages, in Besso's hand, are written on the verso of a letter from C.-E. Guye to Einstein, 31 May 1913, now in a private collection), p.[1] with minor rust markings, otherwise in fine condition for a working scientific manuscript.
Publication: Einstein, *Collected Papers*, 4:344-359 (introduction), 4:360-473 (transcription, with notes), and 4:630-682 (facsimile).

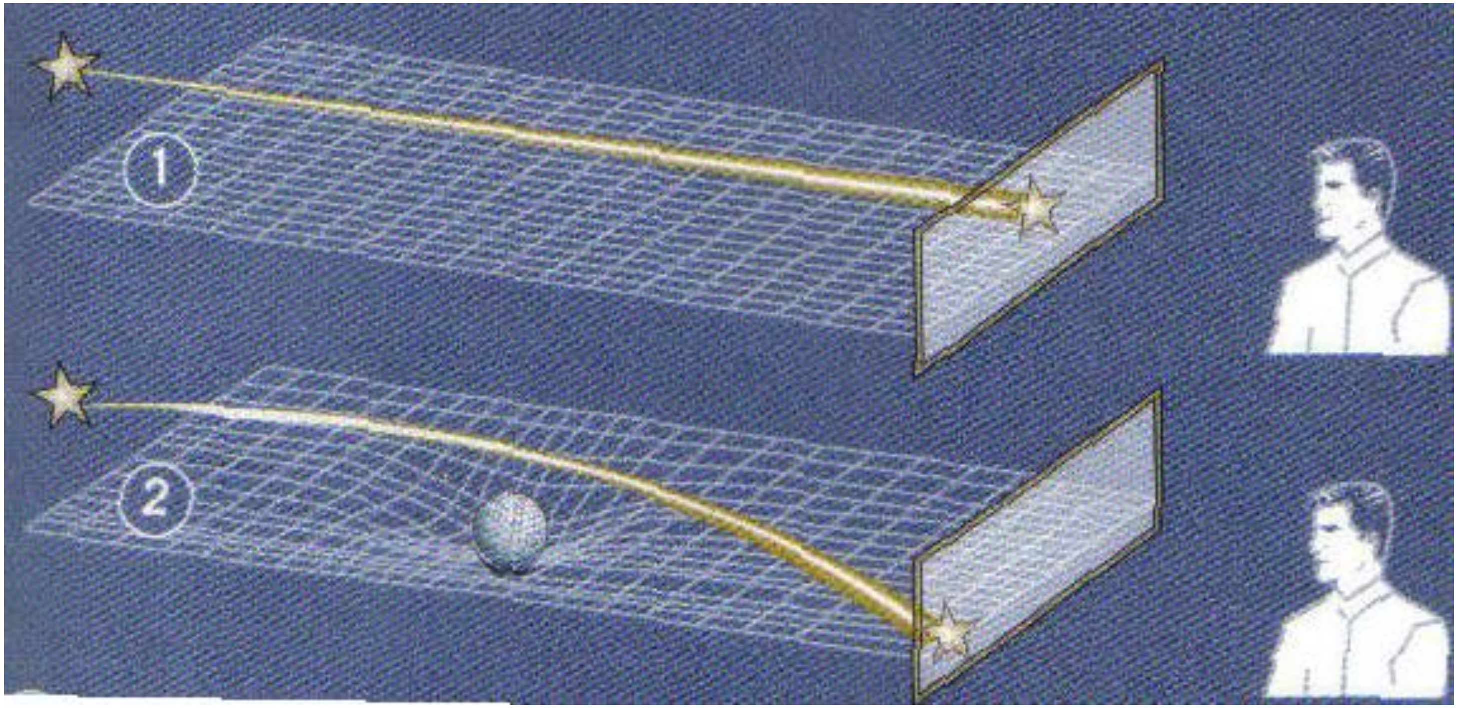
Bespoke box by Alain Taral, in Macassar ebony, each sheet in its individual folder.

Provenance: Christie's New York, 25 November 1996, single lot sale – Harvey Plotnick; his sale at Christie's New York, 4 October 2002, lot 81 – acquired at the sale by the present owner.

[Chinese translation available p32]

Test imagined by Einstein: Effect of gravitation on the propagation of light rays

A new concept of space-time



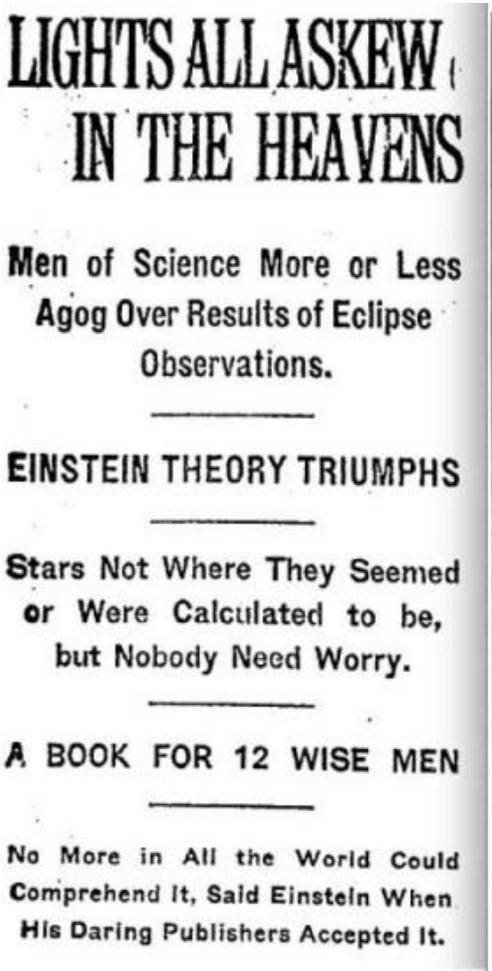
• Gravity is the result of distortions in space-time created by mass and energy

• Light follows the geodesics of this **Riemann** space

1919 Expedition: **Arthur Eddington**

A wonderful story ...

Effect of gravitation on the propagation of light rays



LIGHTS ALL ASKEW IN THE HEAVENS

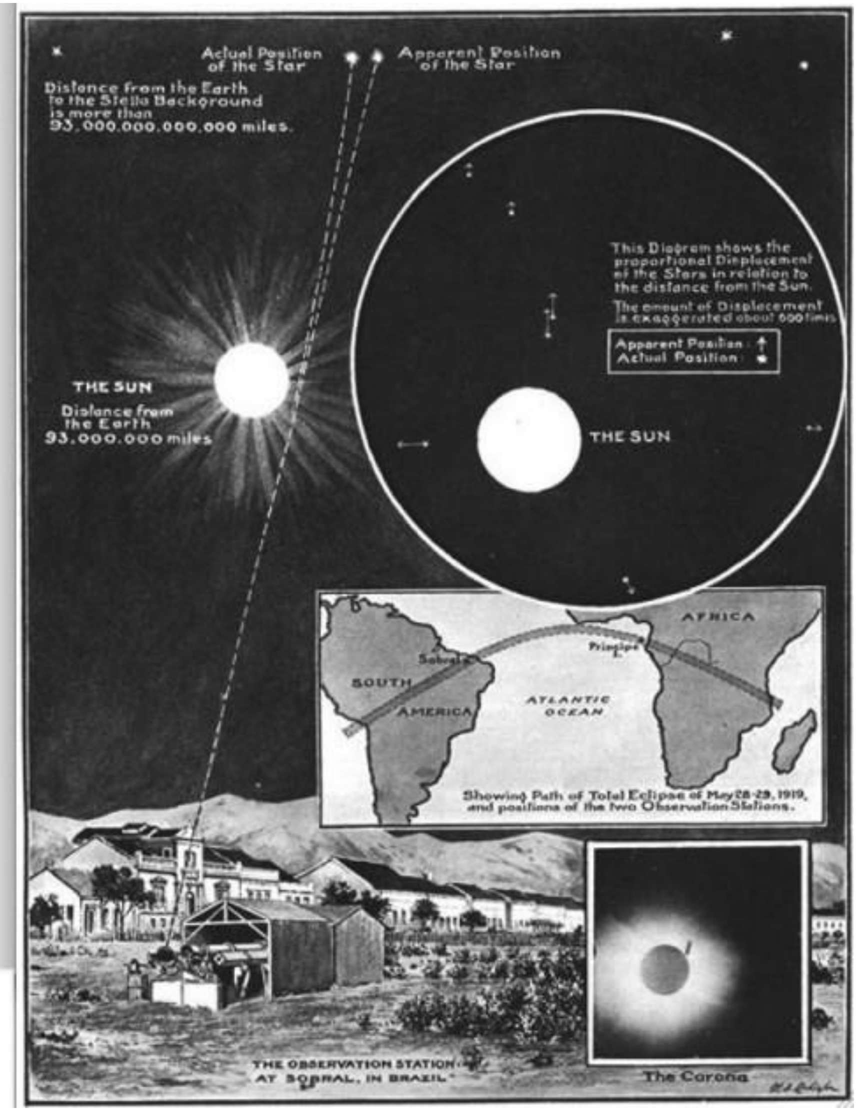
Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

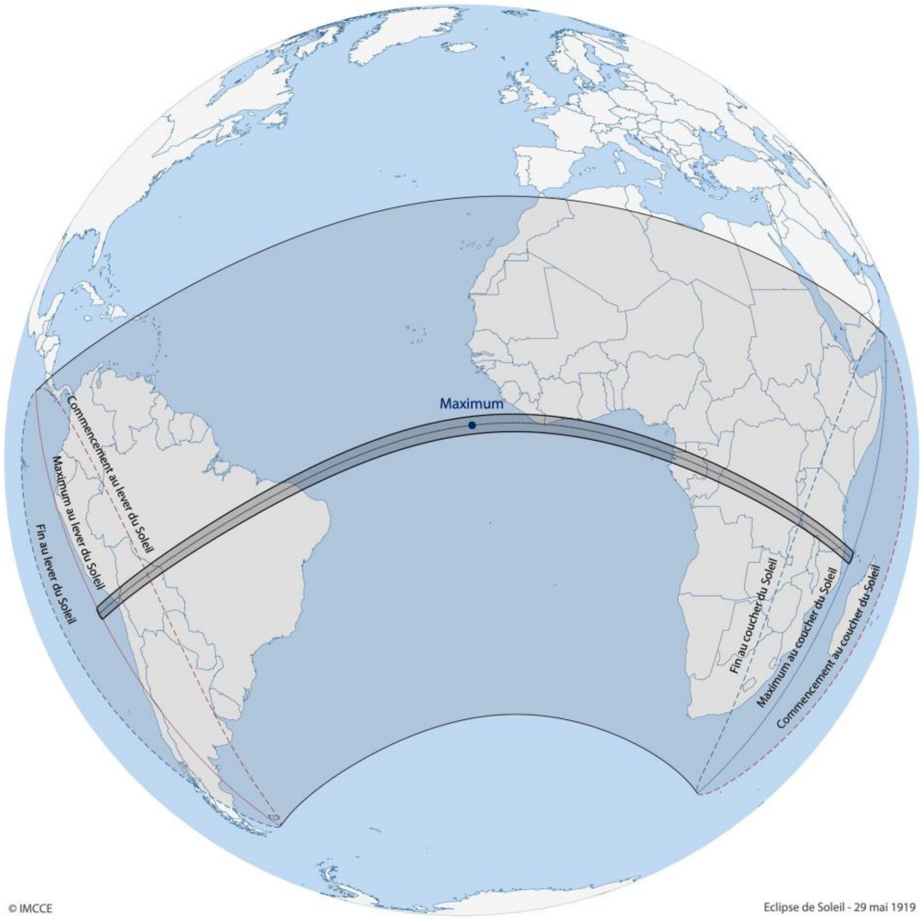


Results of the May 29, 1919 eclipse reported in the press. New York Times, November 10, 1919 (left); Illustrated London News, November 22, 1919 (right).

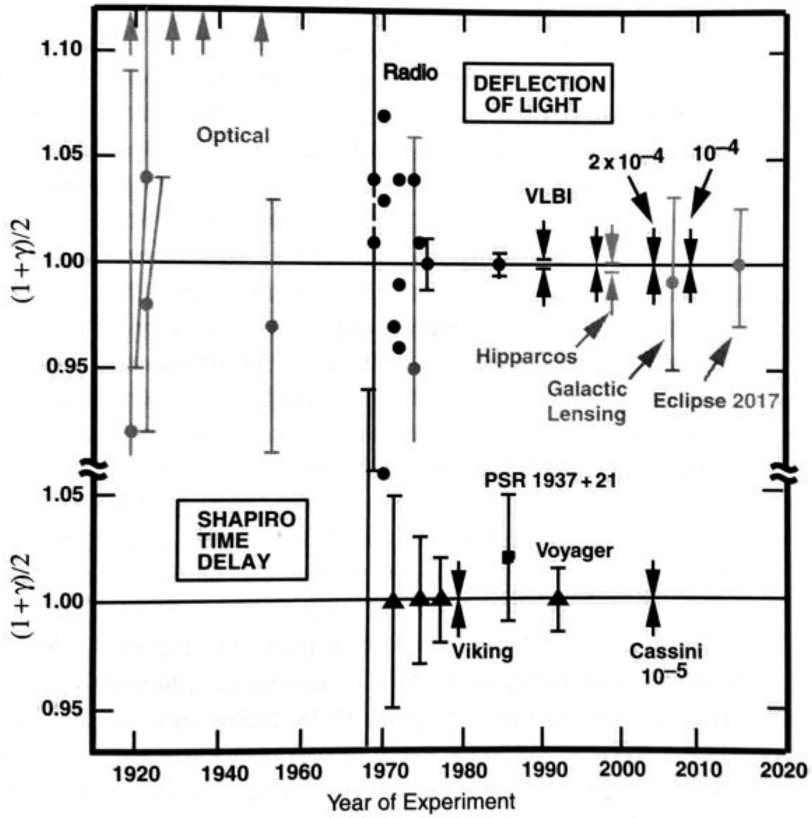
Effect of gravitation on the propagation of light rays

https://lastronomieafrique.com/des-observations-sur-lile-de-principe-confirment-la-theorie-de-la-relativite-generale-deinstein-des-2019/

Très bon article !



Effect of gravitation on the propagation of light rays



$$\delta\theta \approx 1.98 \text{ arcseconds}$$

$$1 \text{ arcsecond} = 1'' = 1^\circ/3600$$

Clifford M. Will, p 161

- Mesurements of the coefficient $(1+\gamma)/2$ from light deflection and time delay mesurements. Its GR value is unity. The arrows at the top denote anomalously large value from early eclipse expeditions. The Shapiro time-delay measurements using the Cassini spacecraft yielded an agreement with GR to 10^{-3} percent, and VLBI (Very Long Baseline Interferometry) light deflection measurements have reached 0.01 percent. Hipparcos denotes the optical astrometry satellite, which reached 0.2 percent.

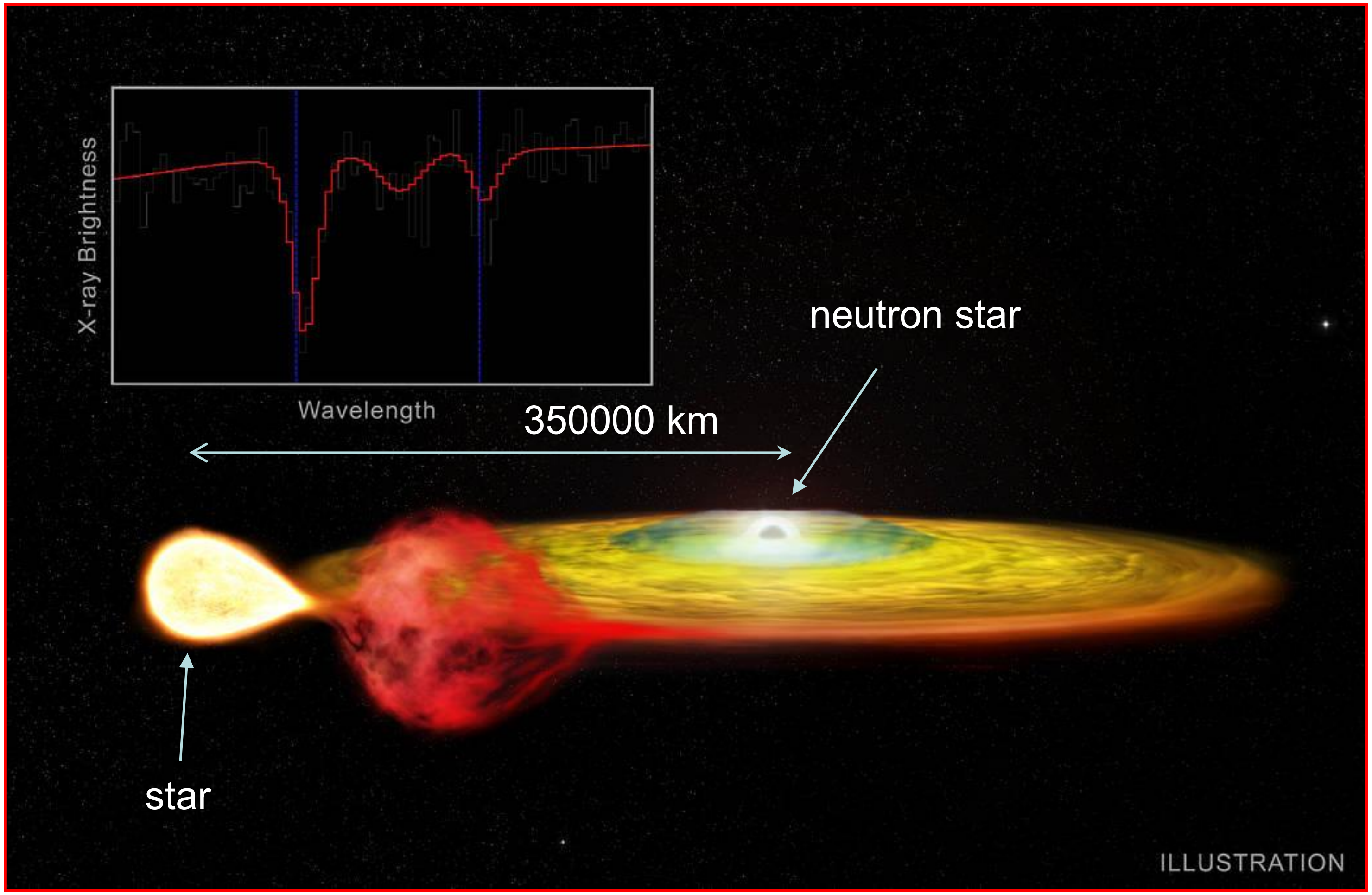
Test imagined by Einstein: **gravitational redshift**

What do Albert Einstein, the Global Positioning System (GPS), and a pair of stars 200,000 trillion miles from Earth have in common?

- The answer is an effect from Einstein's General Theory of Relativity called the "**gravitational redshift**," where light is shifted to redder colors because of gravity. Using NASA's Chandra X-ray Observatory, astronomers have discovered the phenomenon in two stars orbiting each other in our galaxy about 29,000 light-years (200,000 trillion miles) away from Earth. While these stars are very distant, gravitational redshifts have tangible impacts on modern life, as scientists and engineers **must take them into account to enable accurate positions for GPS**.
- While scientists have found incontrovertible evidence of gravitational redshifts in our solar system, it has been challenging to observe them in more distant objects across space. The new Chandra results provide convincing evidence for gravitational redshift effects at play in a new cosmic setting.
- The intriguing system known as 4U 1916-053 contains two stars in a remarkably close orbit. One is the core of a star that has had its outer layers stripped away, leaving a star that is much denser than the Sun. The other is a neutron star, an even denser object created when a massive star collapses in a supernova explosion. The neutron star (grey) is shown in this artist's impression at the center of a disk of hot gas pulled away from its companion (white star on left).

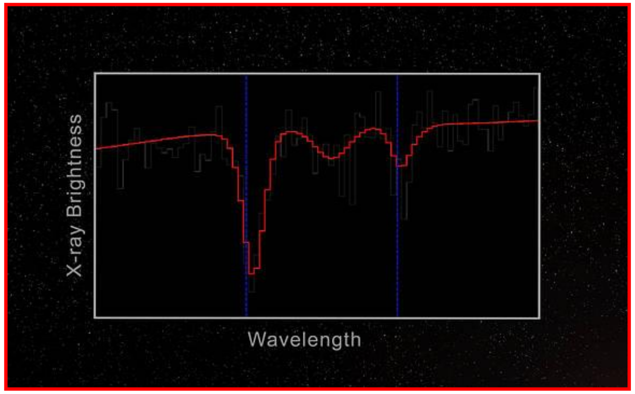
- https://www.nasa.gov/mission_pages/chandra/images/einstein-s-theory-of-relativity-critical-for-gps-seen-in-distant-stars.html

Test imagined by Einstein: gravitational redshift



Test imagined by Einstein: **gravitational redshift**

- These two compact stars are only about 215,000 miles apart, roughly the distance between the Earth and the Moon. While the Moon orbits our planet once a month, the dense companion star in 4U 1916-053 whips around the neutron star and completes a full orbit in only 50 minutes.
- In the new work on 4U 1916-053, the team analyzed X-ray spectra -- that is, the amounts of X-rays at different wavelengths -- from Chandra. They found the characteristic signature of the absorption of X-ray light by iron and silicon in the spectra. In three separate observations with Chandra, the data show a sharp drop in the detected amount of X-rays close to the wavelengths where the iron or silicon atoms are expected to absorb the X-rays. One of the spectra showing absorption by iron is included in the main graphic, and an additional graphic shows a spectrum with absorption by silicon.
- However, the wavelengths of these characteristic signatures of iron and silicon were shifted to longer, or redder wavelengths compared to the laboratory values found here on Earth (shown with the dashed line). The researchers found that the shift of the absorption features was the same in each of the three Chandra observations, and that it was too large to be explained by motion away from us. Instead they concluded it was caused by gravitational redshift.

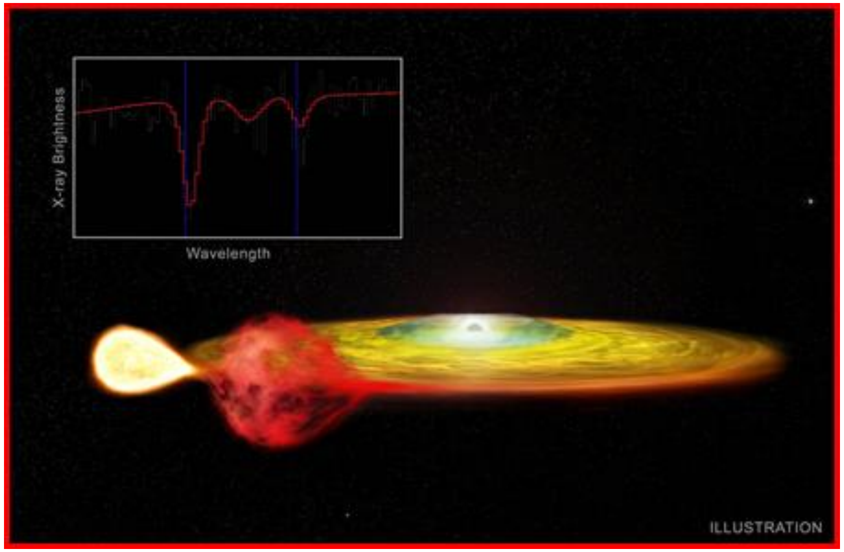


Test imagined by Einstein: **gravitational redshift**

- How does this connect with General Relativity and GPS? As predicted by Einstein's theory, clocks under the force of gravity run at a slower rate than clocks viewed from a distant region experiencing weaker gravity. This means that clocks on Earth observed from orbiting satellites run at a slower rate. To have the high precision needed for GPS, this effect needs to be taken into account or there will be small differences in time that would add up quickly, calculating inaccurate positions.
- All types of light, including X-rays, are also affected by gravity. An analogy is that of a person running up an escalator that is going down. As they do this, the person loses more energy than if the escalator was stationary or going up. The force of gravity has a similar effect on light, where a loss in energy gives a lower frequency. Because light in a vacuum always travels at the same speed, the loss of energy and lower frequency means that the light, including the signatures of iron and silicon, shift to longer wavelengths.
- This is the first strong evidence for absorption signatures being shifted to longer wavelengths by gravity in a pair of stars that has either a neutron star or black hole. Strong evidence for gravitational redshifts in absorption has previously been observed from the surface of white dwarfs, with wavelength shifts typically only about 15% of that for 4U 1916-053.

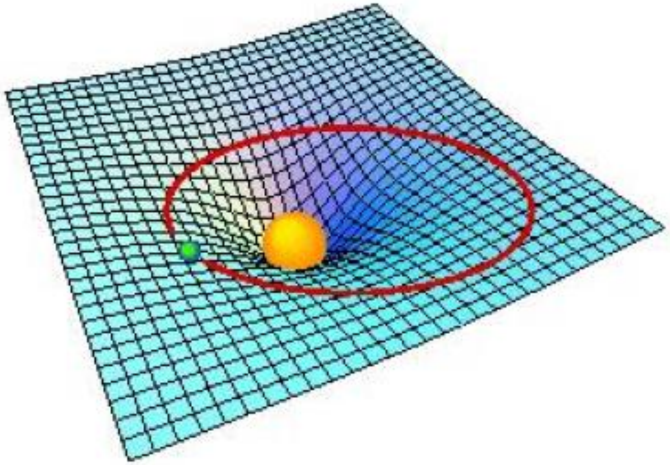
Test imagined by Einstein: **gravitational redshift**

- Scientists say it is likely that a gaseous atmosphere blanketing the disk near the neutron star (shown in blue) absorbed the X-rays, producing these results. The size of the shift in the spectra allowed the team to calculate how far this atmosphere is away from the neutron star, using General Relativity and assuming a standard mass for the neutron star. They found that the atmosphere is located 1,500 miles from the neutron star, about half the distance from Los Angeles to New York and equivalent to only 0.7% of the distance from the neutron star to the companion. It likely extends over several hundred miles from the neutron star.
- In two of the three spectra there is also evidence for absorption signatures that have been shifted to even redder wavelengths, corresponding to a distance of only 0.04% of the distance from the neutron star to the companion. However, these signatures are detected with less confidence than the ones further away from the neutron star.



Predictions of the theory of GR: **Gravitational waves**

Gravitation = curvature of the space-time



Gravitational wave = small disturbance propagating at the speed of light

Predictions of the theory of GR: **Gravitational waves**

- The existence of gravitational waves is predicted by the theory of general relativity: masses in motion relative to one another could lose part of their energy, which would be emitted into space **in the form of waves with a speed close to that of light**.

- Possible sources of gravitational waves include **supernova explosions**, **black holes** acting on each other, **pulsars** (rotating neutron stars) and **binary systems of neutron stars**, when their components melt and then disappear.

↓

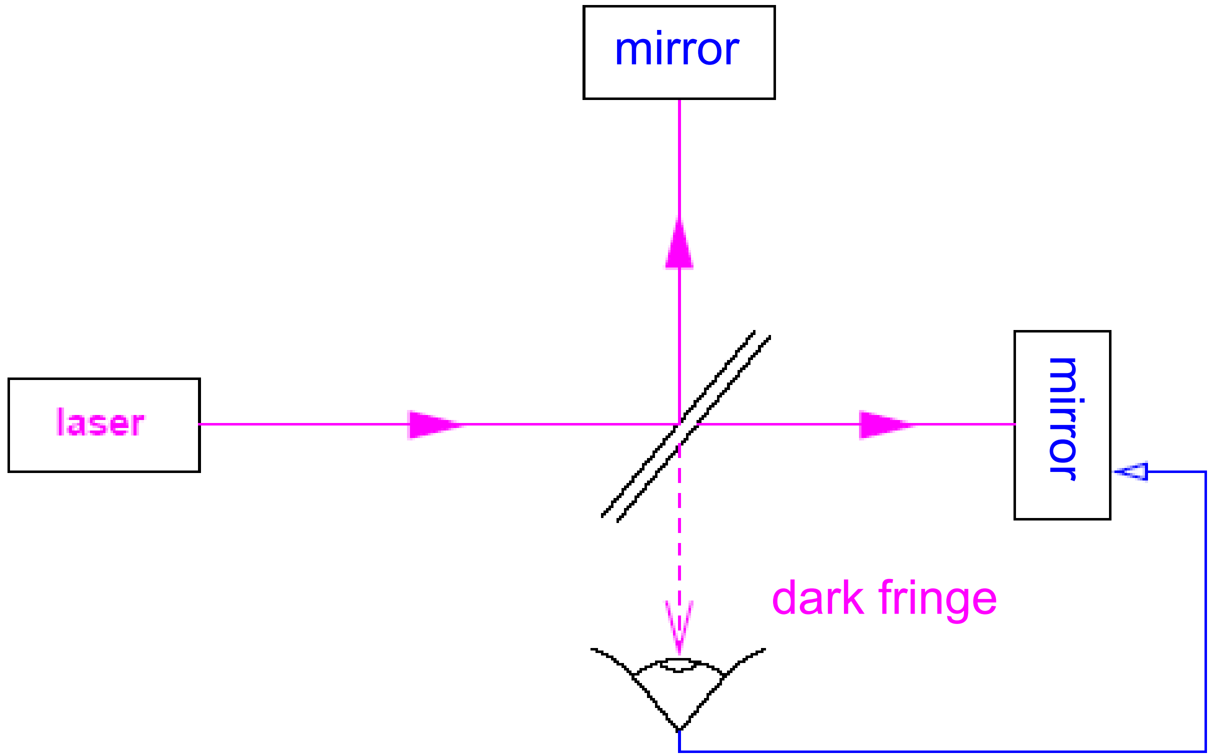
- Several teams involving thousands of researchers have been tracking gravitational waves for decades. Two major detection instruments have been developed: LIGO (Laser Interferometer Gravitational-Wave Observatory) in the USA, and Virgo in Europe (in Italy, near Pisa). **The first detection of gravitational waves was made on September 14, 2015 by LIGO, with the help of the Virgo team for data analysis.**

Predictions of the theory of GR:
Gravitational waves

- The merger of two neutron stars was detected on August 17, 2017 in the galaxy NGC 4993, both as gravitational waves and as light. In all, in addition to **LIGO** and **Virgo**, some 70 observatories on the ground and in space took part in monitoring the event. This is the first time that gravitational waves have been detected with an electromagnetic counterpart. This detection reinforces the hypothesis that gamma-ray bursts, or at least some of them, are the result of the merger of two neutron stars. This detection verified with **an accuracy of one part in 10^{15} GR's prediction** that the two signals are moving at the same speed, thus ruling out a large number of other theories which gave different predictions.

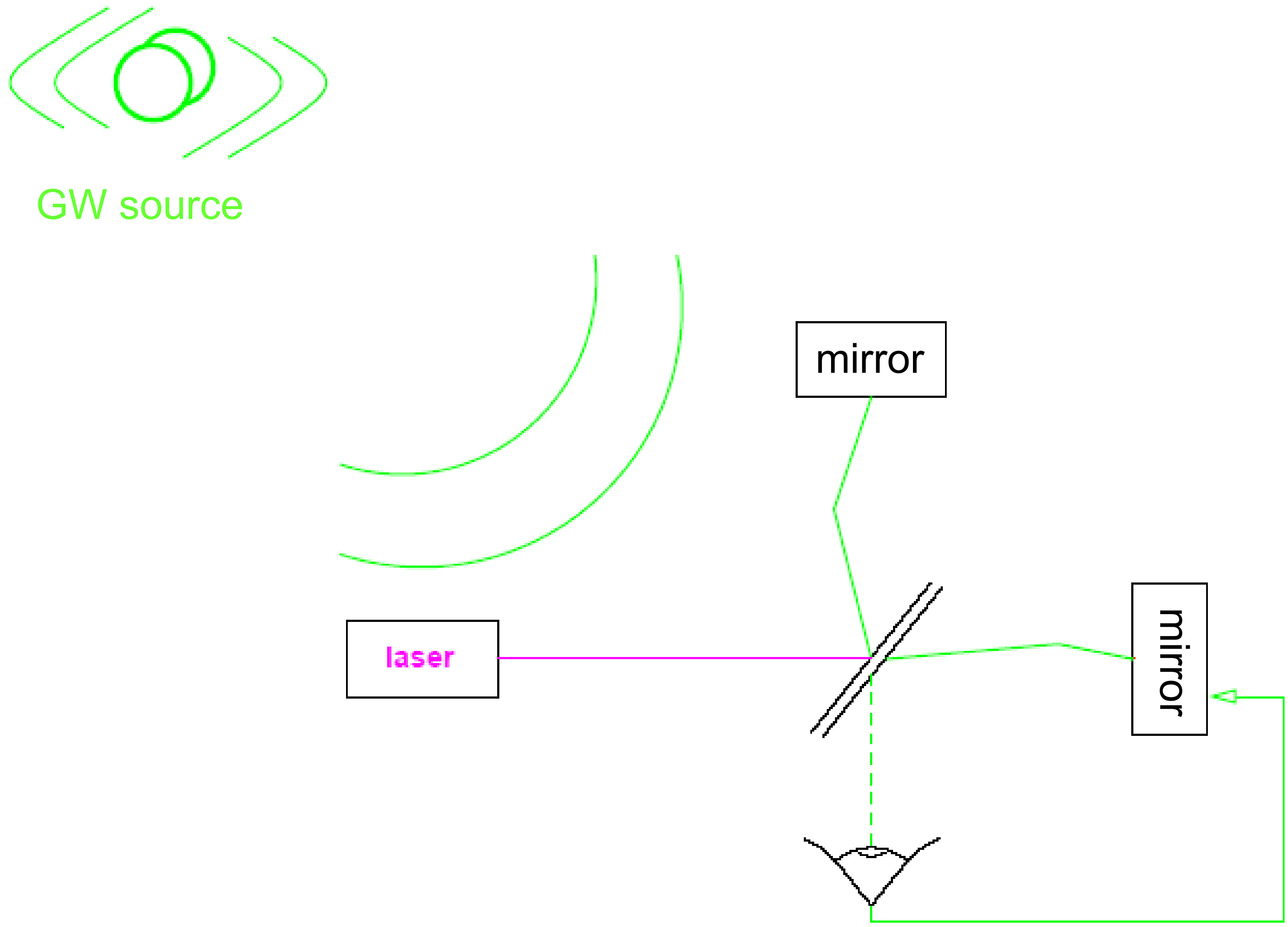
Gravitational waves detection

Interferometric gravitational-wave detector = **Michelson interferometer**



- Interferometry is a family of techniques in which waves, generally electromagnetic waves, are superimposed, causing the phenomenon of interference in order to extract information.

Gravitational waves detection

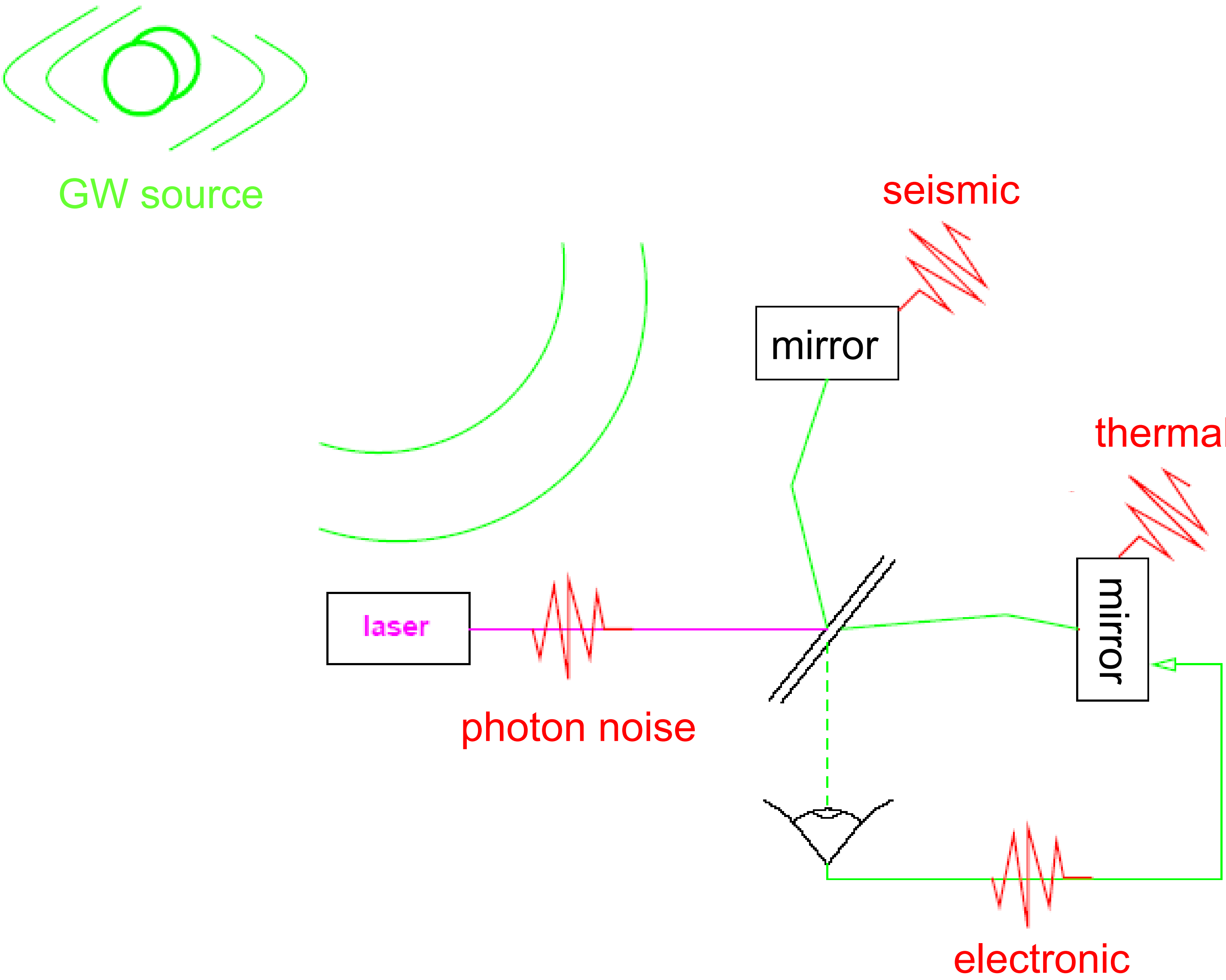


precise measurement of $\Delta L = L_2 - L_1$

detects phase change

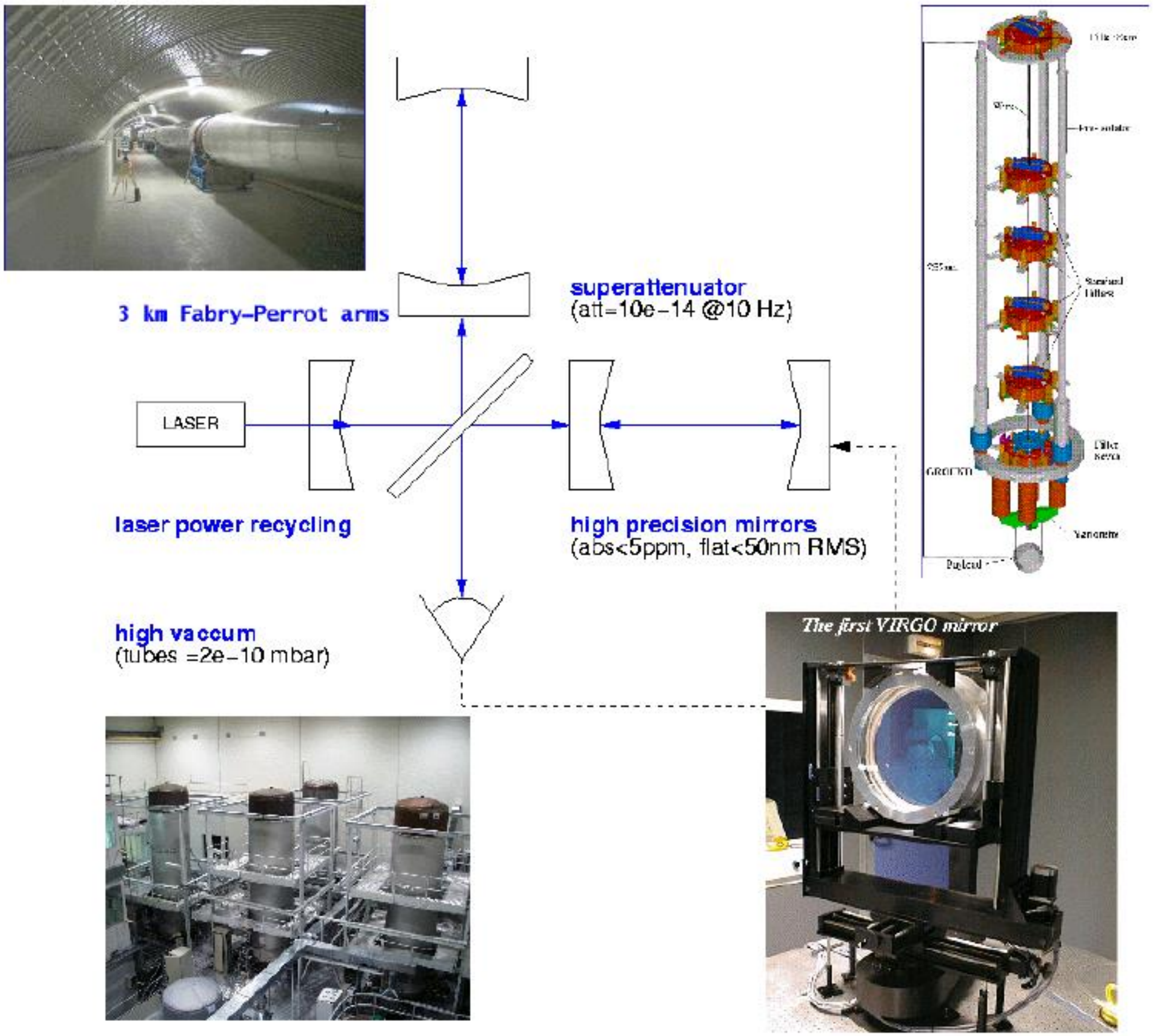
Gravitational waves detection

GW detection: **noise sources to combat**



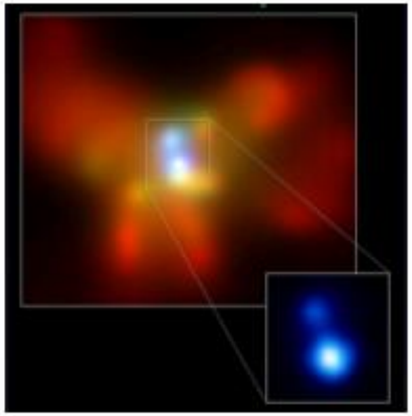
Gravitational waves detection

Technological challenges

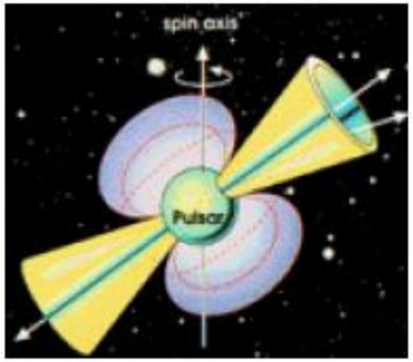


Gravitational waves detection

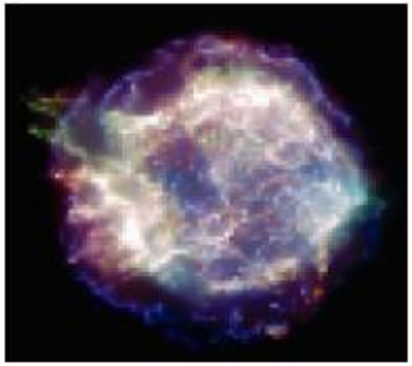
Astrophysical sources



Compact object binaries (a black hole or a neutron star orbiting a non-degenerate stellar companion)



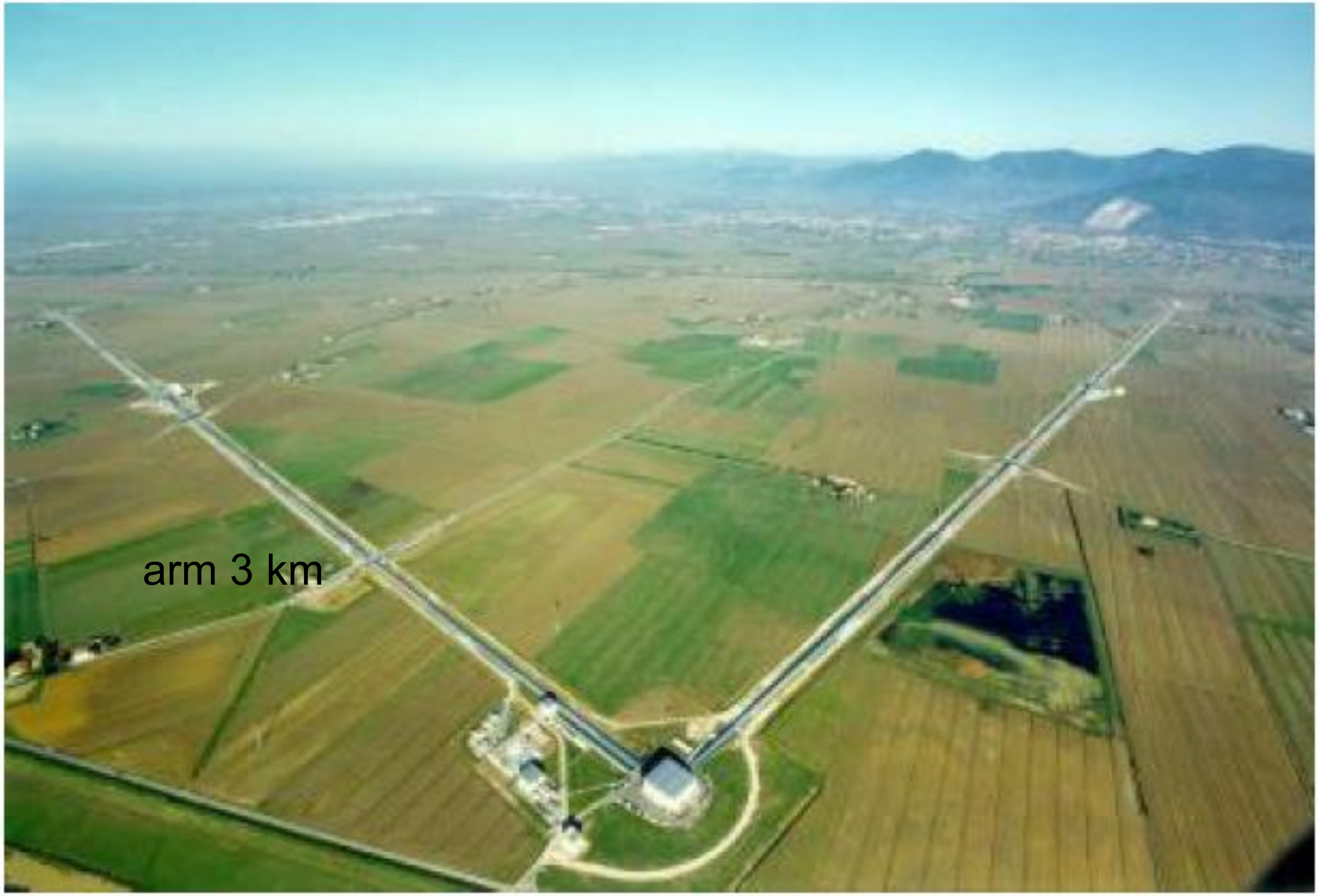
Neutron stars



Supernovae

Gravitational waves detection

→ Fundamental research leads to major technological breakthroughs!



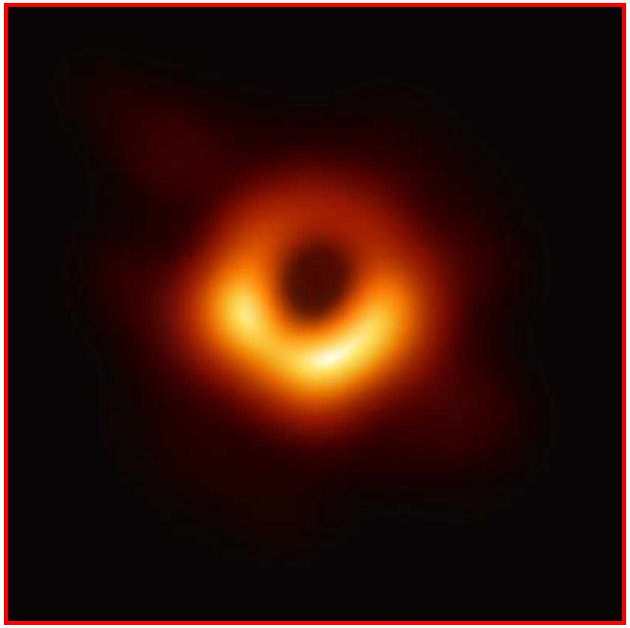
Location: Cascina, near Pisa, Italy

Application of GR: **black holes**

A **black hole** is a region of spacetime where gravity is so strong that nothing, including light or other electromagnetic waves, has enough energy to escape it. **The theory of general relativity predicts** that a sufficiently compact mass can deform spacetime to form a black hole.

- Messier 87 (also known as Virgo A or NGC 4486, generally abbreviated to M87) is a supergiant elliptical galaxy in the constellation Virgo.

- M87 is about 16.4 million parsecs (53 million light-years=$5x10^{20}$ km) from Earth.



Direct image of a supermassive black hole at the core of Messier 87.