Problem Set 5
Transport phenomena in semiconductors

1 Nonequilibrium carrier distributions

Under nonequilibrium and nonstationary conditions, the electron and hole densities in a semiconductor, denoted $n(\mathbf{r}, t)$ and $p(\mathbf{r}, t)$ respectively, depend on space ($\mathbf{r}$) and time (t). In the presence of both an electric field $\boldsymbol{\mathcal{E}}$ and a density gradient, the electron and hole current densities are given by

$$\mathbf{J}_n = en\mu_n\boldsymbol{\mathcal{E}} + eD_n\nabla n,$$
$$\mathbf{J}_p = ep\mu_p\boldsymbol{\mathcal{E}} - eD_p\nabla p,$$

where $e > 0$ is the elementary charge, μ_n (μ_p) the electron (hole) mobility, and D_n (D_p) the diffusion coefficient for electrons (holes). The densities are related to the current densities through the continuity equations

$$\frac{\partial n}{\partial t} = +\frac{1}{e}\nabla\cdot\mathbf{J}_n + G_n - R_n,$$
$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla\cdot\mathbf{J}_p + G_p - R_p.$$

Here, $G_n = G_n(\mathbf{r}, t)$ [$R_n = R_n(\mathbf{r}, t)$] is the generation [recombination] rate, *i.e.*, the number of electrons generated [substracted] in the conduction band per unit time and unit volume. Similarly, the rates G_p and R_p refer to the holes generated or substracted in the valence band.

(a) Show that the partial differential equations determining n and p are given by

$$\frac{\partial n}{\partial t} = D_n\nabla^2 n + \mu_n\nabla\cdot(n\boldsymbol{\mathcal{E}}) + G_n - R_n, \tag{1.1}$$
$$\frac{\partial p}{\partial t} = D_p\nabla^2 p - \mu_p\nabla\cdot(p\boldsymbol{\mathcal{E}}) + G_p - R_p. \tag{1.2}$$

(b) Although it may not be directly apparent, Eqs. (1.1) and (1.2) are coupled. Why is that so?

2 Generation and recombination of electron-hole pairs in doped semiconductors

We now consider a semiconductor doped with donor or acceptor impurities, which are assumed to be all ionized. We further assume that the kinetics of the carriers is solely determined by radiative generation-recombination processes of electron-hole pairs. We call G_{eh} and R_{eh} the corresponding rates.

(a) With the help of the doped semiconductor bandstructure (assumed, in the effective mass approximation, to be parabolic and with a direct gap), sketch the two following radiative processes:

- An electron in the conduction band makes an optical transition to the valence band (annihilation of an electron-hole pair) with the emission of a photon.
- An electron from the valence band is transfered to the conduction band (creation of an electron-hole pair) with absorption of a photon.

Convince yourself that for these processes, $G_n = G_p = G_{\text{eh}}$ and $R_n = R_p = R_{\text{eh}}$.

(b) Does G_{eh} depends on n? On p? On temperature?

(c) Same question for R_{eh}. Argue that it is reasonable to assume that $R_{\text{eh}} \propto np$.

(d) From now on, we assume that

$$R_{\text{eh}} - G_{\text{eh}} = B\,(np - n_0 p_0)\,, \tag{2.1}$$

where n_0 and p_0 are the equilibrium electron and hole densities, and where $B = B(T)$ is a constant that depends essentially on temperature and on the material under consideration. Briefly justify expression (2.1).

(e) Let us consider an n-type semiconductor, with $n_0 \gg p_0$. We also assume that the deviations $\Delta n = n - n_0$ and $\Delta p = p - p_0$ are much smaller than n_0 (*low injection regime*). Show that in this case, Eq. (2.1) reduces to

$$R_{\text{eh}} - G_{\text{eh}} \simeq \frac{p - p_0}{t_p},$$

with $t_p = 1/Bn_0$. What is the dimension of t_p? Note that in this case, Eq. (1.2) yields the following continuity equation for holes minority carriers in an n-type semiconductor:

$$\frac{\partial p}{\partial t} = D_p \nabla^2 p - \mu_p \nabla \cdot (p\boldsymbol{\mathcal{E}}) - \frac{p - p_0}{t_p}. \tag{2.2}$$

If there are no diffusion and no electric field, what is the solution to Eq. (2.2)? Give then a physical interpretation of t_p.

(f) Let us consider now a p-type semiconductor, with $p_0 \gg n_0$. Using similar considerations as in Question (e), show that the continuity equation for electrons minority carriers in a p-type semiconductor is given by

$$\frac{\partial n}{\partial t} = D_n \nabla^2 n + \mu_n \nabla \cdot (n\boldsymbol{\mathcal{E}}) - \frac{n - n_0}{t_n}$$

and give the expression of t_n.

3 Injection of minority carriers at one end of an n-type semiconductor in steady state conditions[1]

Consider a bar of homogeneously doped n-type semiconductor and suppose that an external source (for instance light irradiation or particle bombardment) maintains an excess of holes $\Delta p(x) = \Delta p(0)$, and an excess of electrons $\Delta n(x) = \Delta n(0) = \Delta p(0)$ for any $x \leqslant 0$. Carriers then diffuse in the $x > 0$ part of the semiconductor. In this exercice, we wish to determine the excess densities $\Delta p(x)$ and $\Delta n(x)$ in stationary conditions as well as the internal electric field, that automatically sets in and accompanies the diffusion process.

(a) Let us ignore momentarily the internal electric field which may accompany the diffusion processes. Show that in steady-state conditions, $\Delta p(x)$ is determined by the second-order differential equation

$$D_p \frac{\mathrm{d}^2 \Delta p}{\mathrm{d}x^2} - \frac{\Delta p}{t_p} = 0. \tag{3.1}$$

(b) Solve Eq. (3.1) using appropriate boundary conditions, and express your result as a function of the minority carrier diffusion length $L_p = \sqrt{D_p t_p}$.

[1] © G. Grosso, G. Pastori Parravicini, *Solid State Physics* (Academic Press, 2014)

(c) Estimate L_p for the typical values $D_p = 10\,\text{cm}^2/\text{s}$ and $t_p = 10^{-5}\,\text{s}$.

(d) Determine the corresponding hole diffusion current $J_p^{(\text{diff})}(x)$.

(e) Use the fact that $\Delta n \simeq \Delta p$ (why is that so?) to show that the electron diffusion current is given by $J_n^{(\text{diff})}(x) = -bJ_p^{(\text{diff})}(x)$, with b a constant to be determined.

(f) Use the fact that the total current (diffusion + drift) must vanish (justification?) to show that the electric field is given by

$$\mathcal{E}(x) = \frac{1}{\sigma_n}(b-1)eD_p\frac{\Delta p(0)}{L_p}\,\mathrm{e}^{-x/L_p},$$

where $\sigma_n = en_0\mu_n$ is the conductivity of the n-type semiconductor, and discuss this result.