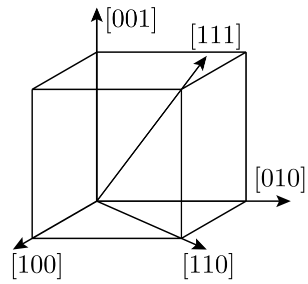
Problem Set 10
Crystalline anisotropy energy and Bloch domain walls



Iron crystals have cubic symmetry. The directions of easy magnetization are the crystalline directions [100], [010], and [001]. We denote α, β, and γ the directional cosines of the magnetization with respect to these directions (the directional cosine of a vector is the cosine of the angle between the vector and the direction). The energy density of crystalline anisotropy is expanded in a power series of these cosines as

$$W(\alpha, \beta, \gamma) = \sum_{p,q,r} A_{p,q,r}\, \alpha^p \beta^q \gamma^r,$$

where the sum is running over integers $p, q, r \geq 0$. The cubic crystal symmetry allows to simplify the general expression. Including terms of order $p + q + r \leq 6$, W can be written in the form

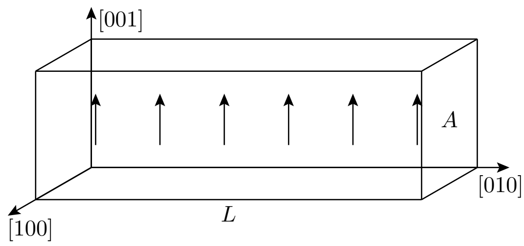
$$W(\alpha, \beta, \gamma) = K_0 + K_1 \left(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2\right) + K_2\, \alpha^2\beta^2\gamma^2,$$

with positive coefficients K_1 and K_2.

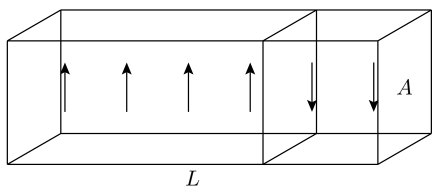
We assume a simple cubic lattice with lattice constant a, and take the ferromagnetic exchange interaction energy between two magnetic moments $\vec{\mu}_i$ and $\vec{\mu}_j$ on nearest neighbor sites as $-J \cos\theta_{ij}$, with $J > 0$ and θ_{ij} the angle between the magnetic moments. The exchange energy between sites of larger distance is neglected.

(a) Take the case when the magnetization is in the plane perpendicular to the direction [010], and at the angle φ with the direction [001]. Express the energy density of magnetic anisotropy $W(\varphi)$ as a function of φ and the coefficients K_0 and K_1. Plot W as a function of φ and comment on the result.

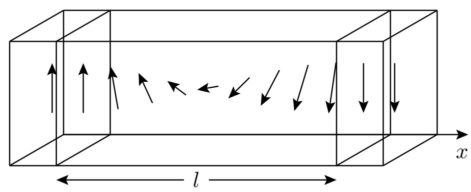
(b) Magnetized at saturation in the direction [001], a monocrystalline iron bar of length L and section A forms a magnetic monodomain. Calculate the crystalline anisotropy energy and the exchange energy of the bar, and give its total magnetic energy U_1.



(c) The iron bar now comprises two magnetic domains whose magnetizations are opposite and saturated, with an abrupt interface. Write the magnetic energy U_2 of the bar in the form $U_2 = U_1 + \Delta U$ and give the expression of ΔU.



(d) We now have a Bloch domain wall of thickness l between the magnetic domains of opposite saturated magnetization. In the Bloch wall, the magnetization stays in the plane perpendicular to [010], and rotates by a constant angle when moving from a crystal plane to the next one in the [010]-direction. From $x = x_0 - l/2$ to $x = x_0 + l/2$, the magnetization rotates progressively by the total angle π. Show that, when $l \gg a$, the energy of the bar can be written as



$$U_{\text{DW}} = \left(K_0 - \frac{3J}{a^3}\right) AL + \left(\frac{K_1 l}{8} + \frac{\pi^2 J}{2al}\right) A.$$

(e) Determine the thickness l of the Bloch domain wall that can be expected in such a bar, and deduce the energy difference $\delta U = U_{\mathrm{DW}} - U_1$.

(f) Using the values $K_1 = 4 \times 10^4\ \mathrm{J/m^3}$, $J = 2 \times 10^{-21}\ \mathrm{J}$ and $a = 2.9\ \text{Å}$, calculate the thickness of the wall and the energies per unit area $\Delta U/A$ and $\delta U/A$. Discuss the result.