Formulaire

Notations

— un champ de vecteur est noté en gras : $\mathbf{v}$;
— un vecteur unitaire est noté avec un chapeau : $\hat{\mathbf{v}} = \mathbf{v}/|\mathbf{v}| = \mathbf{v}/v$;
— le produit vectoriel de deux vecteurs $\mathbf{A}$ et $\mathbf{B}$ est noté $\mathbf{A} \times \mathbf{B}$.

Systèmes de coordonnées

Coordonnées cartésiennes

$$\mathrm{d}\mathbf{l} = \mathrm{d}x\,\hat{\mathbf{x}} + \mathrm{d}y\,\hat{\mathbf{y}} + \mathrm{d}z\,\hat{\mathbf{z}}; \qquad \mathrm{d}\tau = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$\begin{aligned}
\nabla f &= \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} \\
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\nabla \times \mathbf{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}} \\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}$$

Coordonnées sphériques

$$\begin{aligned}
x &= r\sin\theta\cos\varphi & \hat{\mathbf{x}} &= \sin\theta\cos\varphi\,\hat{\mathbf{r}} + \cos\theta\cos\varphi\,\hat{\boldsymbol{\theta}} - \sin\varphi\,\hat{\boldsymbol{\varphi}} \\
y &= r\sin\theta\sin\varphi & \hat{\mathbf{y}} &= \sin\theta\sin\varphi\,\hat{\mathbf{r}} + \cos\theta\sin\varphi\,\hat{\boldsymbol{\theta}} + \cos\varphi\,\hat{\boldsymbol{\varphi}} \\
z &= r\cos\theta & \hat{\mathbf{z}} &= \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}
\end{aligned}$$

$$\begin{aligned}
r &= \sqrt{x^2 + y^2 + z^2} & \hat{\mathbf{r}} &= \sin\theta\cos\varphi\,\hat{\mathbf{x}} + \sin\theta\sin\varphi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\
\theta &= \arctan(\sqrt{x^2 + y^2}/z) & \hat{\boldsymbol{\theta}} &= \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\
\varphi &= \arctan(y/x) & \hat{\boldsymbol{\varphi}} &= -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}}
\end{aligned}$$

$$\mathrm{d}\mathbf{l} = \mathrm{d}r\,\hat{\mathbf{r}} + r\mathrm{d}\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\mathrm{d}\varphi\,\hat{\boldsymbol{\varphi}}; \qquad \mathrm{d}\tau = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

$$\begin{aligned}
\nabla f &= \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} \\
\nabla \cdot \mathbf{v} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\,v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} \\
\nabla \times \mathbf{v} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\,v_\varphi) - \frac{\partial v_\theta}{\partial \varphi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r}(r v_\varphi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\varphi}} \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}
\end{aligned}$$

Coordonnées cylindriques

$$\begin{aligned} x &= r\sin\theta \\ y &= r\cos\theta \\ z &= z \end{aligned} \qquad \begin{aligned} \hat{\mathbf{x}} &= \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \\ \hat{\mathbf{y}} &= \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\boldsymbol{\theta}} \\ \hat{\mathbf{z}} &= \hat{\mathbf{z}} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ \theta &= \arctan(y/x) \\ z &= z \end{aligned} \qquad \begin{aligned} \hat{\mathbf{r}} &= \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\theta}} &= -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} &= \hat{\mathbf{z}} \end{aligned}$$

$$\mathrm{d}\mathbf{l} = \mathrm{d}r\,\hat{\mathbf{r}} + r\mathrm{d}\theta\,\hat{\boldsymbol{\theta}} + \mathrm{d}z\,\hat{\mathbf{z}}; \qquad \mathrm{d}\tau = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} \\ \nabla\cdot\mathbf{v} &= \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z} \\ \nabla\times\mathbf{v} &= \left[\frac{1}{r}\frac{\partial v_z}{\partial\theta} - \frac{\partial v_\theta}{\partial z}\right]\hat{\mathbf{r}} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\mathbf{z}} \\ \nabla^2 f &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2} \end{aligned}$$

Théorèmes fondamentaux

Théorème du gradient : $\int_{\mathbf{a}}^{\mathbf{b}} \mathrm{d}\mathbf{l}\cdot\nabla f = f(\mathbf{b}) - f(\mathbf{a})$

Théorème de la divergence : $\int_{\mathcal{V}} \mathrm{d}\tau\,(\nabla\cdot\mathbf{A}) = \oint_{\mathcal{S}} \mathrm{d}\mathbf{a}\cdot\mathbf{A}$ (Green-Ostrogradski)

Théorème du rotationnel : $\int_{\mathcal{S}} \mathrm{d}\mathbf{a}\cdot(\nabla\times\mathbf{A}) = \oint_{\mathcal{P}} \mathrm{d}\mathbf{l}\cdot\mathbf{A}$ (Stokes)

Constantes fondamentales

$\epsilon_0 = 8.85\times10^{-12}\,\mathrm{C^2/Nm^2}$ (permittivité du vide)

$\mu_0 = 4\pi\times10^{-7}\,\mathrm{N/A^2}$ (perméabilité du vide)

$c = 3.00\times10^{8}\,\mathrm{m/s}$ (vitesse de la lumière)

$e = 1.60\times10^{-19}\,\mathrm{C}$ (charge de l'électron)

$m = 9.11\times10^{-31}\,\mathrm{kg}$ (masse de l'électron)