Answers to selected exercises

Exercise 1.1

(a) $A = \sqrt{\lambda}$

(b) $\langle x \rangle = 0$; $\langle x^2 \rangle = 1/2\lambda^2$

(c) $\sigma = 1/\sqrt{2}\lambda$; $|\psi(\pm\sigma)|^2 = \lambda \exp(-\sqrt{2}) \simeq 0.24\lambda$; proba. outside $2\int_\sigma^\infty \mathrm{d}x|\psi|^2 = \exp(-\sqrt{2}) \simeq 0.24$

Exercise 1.3

(a) $A = (2am/\pi\hbar)^{1/4}$

(b) $V(x) = 2ma^2x^2$ (*cf.* harmonic oscillator)

(c) $\langle x \rangle = 0$; $\langle x^2 \rangle = \hbar/4am$; $\langle p \rangle = 0$, $\langle p^2 \rangle = \hbar am$.

(d) $\sigma_x = \sqrt{\hbar/4am}$; $\sigma_p = \sqrt{\hbar am}$; $\sigma_x\sigma_p = \hbar/2$

Exercise 2.2

$\langle x \rangle = 0$; $\langle p \rangle = 0$; $\langle x^2 \rangle = \frac{\hbar}{m\omega}(n + 1/2)$; $\langle p^2 \rangle = \hbar m\omega(n + 1/2)$; $\langle T \rangle = \frac{\hbar\omega}{2}(n + 1/2) = E_n/2$; $\sigma_x\sigma_p = \hbar(n + 1/2) \geqslant \hbar/2$

Exercise 3.1

(a) $\nu > -1/2$

(b) yes; yes; no

Exercise 3.4

(a) $\mathrm{d}\langle Q \rangle/\mathrm{d}t = 0$; (b) $\mathrm{d}\langle H \rangle/\mathrm{d}t = 0$ (conservation of energy); (c) $\mathrm{d}\langle x \rangle/\mathrm{d}t = \langle p \rangle/m$; (d) $\mathrm{d}\langle p \rangle/\mathrm{d}t = -\langle \mathrm{d}V/\mathrm{d}x \rangle$ (Ehrenfest's theorem)