Exam — Session 1

Duration: 2 h.

Documents, cell phones, computers, tablets, pocket calculators, etc., are not allowed.
The text contains 3 pages in total, and the 2 exercises are independent from one another.

1 Charge carrier transport in a doped semiconductor

We consider a semiconductor with equilibrium electron and hole concentrations n_0 and p_0, respectively. In the following, we shall address carrier transport phenomena and the *non-equilibrium* electron and hole concentrations are denoted $n(\mathbf{r}, t)$ and $p(\mathbf{r}, t)$, respectively. The elementary charge is denoted $e\,(> 0)$. We introduce the electron and hole mobilities, μ_n and μ_p, respectively. The diffusion coefficient of electrons and holes are denoted D_n and D_p, respectively.

1.1 Drift and diffusion currents

(a) In the presence of an external electric field $\mathbf{E}$, express the electron and hole *drift* current densities, denoted $\mathbf{J}_\mathrm{n}^\mathrm{drift}$ and $\mathbf{J}_\mathrm{p}^\mathrm{drift}$, respectively. For a given orientation of $\mathbf{E}$, draw the electron and hole fluxes as well as the electron and hole current densities. Connect the electron (hole) mobilities to the electron (hole) conductivities $\sigma_{\mathrm{n(p)}}$.

(b) In the presence of charge carrier concentration gradients, we define the electron and hole *diffusion* current densities $\mathbf{J}_\mathrm{n}^\mathrm{diff}$ and $\mathbf{J}_\mathrm{p}^\mathrm{diff}$, respectively. Give their expressions as a function of $n(\mathbf{r}, t)$ [$p(\mathbf{r}, t)$] and D_n [D_p] and, as in the previous question, qualitatively draw, both for electrons and holes, the charge carrier concentration gradient, the associated particle flux, and the associated diffusion current density.

(c) Write the *total* current density $\mathbf{J}^\mathrm{tot}$.

1.2 Doped semiconductor

We now consider an homogeneously doped semiconductor with an electronic gap $\mathcal{E}_\mathrm{G}$ and we focus on the case of an electron-doped (*i.e.*, n-doped) system, where doping arises from an impurity band, with donor concentration N_d, situated at an energy $\delta\mathcal{E} \ll \mathcal{E}_\mathrm{G}$ below the conduction band minimum.

(a) Give an example of an n-doped semiconductor. Explain qualitatively (no calculations are requested) how n_0 can be determined in the following three temperature regimes: $k_\mathrm{B}T \ll \delta\mathcal{E}$ (*frozen* regime); $k_\mathrm{B}T \gtrsim \delta\mathcal{E}$ (*saturation* regime); $k_\mathrm{B}T \gg \delta\mathcal{E}$ (*intrinsic* regime). The temperature is denoted T and k_B is the Boltzmann constant.

(b) Draw qualitatively $\ln(n_0)$ as a function of $1/T$.

(c) In the following, we shall consider the saturation regime. Express the charge neutrality condition. Knowing that the *intrinsic* charge concentration at room temperature is typically $n_\mathrm{i} \sim 10^{16}\ \mathrm{m}^{-3}$ and that $N_\mathrm{d} \sim 10^{20}\ \mathrm{m}^{-3}$, give a numerical estimate of p_0. Comment on this result and justify why holes are called “minority carriers” in an n-doped semiconductor.

1.3 Continuity equation for minority carriers in an n-doped semiconductor

In the following we consider, for simplicity, a one-dimensional (1d) problem along the x axis.

(a) Write the continuity equation that connects the hole concentration and $J_{\mathrm{p}}^{\mathrm{tot}}$ in the absence of generation and recombination of charge carriers.

(b) We now consider generation and recombination of electron-hole pairs. How can electron-hole pairs be generated and then recombine in a semiconductor? Justify precisely that the difference between the electron-hole pair generation and recombination rates can be written as $\beta\,(n_0 p_0 - np)$, where the (x,t) dependence of n and p is implicitly considered. What is the physical meaning and the dimension of β?

(c) Still in the case of an n-doped semiconductor, demonstrate that one can approximate the difference between the generation and recombination rates as $(p_0 - p)/\tau_{\mathrm{p}}$. What is the physical meaning and the dimension of τ_{p}?

(d) Demonstrate that the continuity equation for holes writes

$$\frac{\partial p}{\partial t} = D_{\mathrm{p}} \frac{\partial^2 p}{\partial x^2} - \mu_{\mathrm{p}} \frac{\partial (pE)}{\partial x} + \frac{p_0 - p}{\tau_{\mathrm{p}}}. \tag{1.1}$$

1.4 Diffusion of minority carriers in a 1d n-doped semiconductor

We consider a 1d n-doped semiconductor in which a constant excess of carriers (electrons *and* holes) $\Delta p_0 = \Delta n_0$ is maintained in the region $x \leqslant 0$, whereas no excess carriers are injected in the region $x > 0$, such that $\lim_{x\to\infty} p(x) = p_0$. For now, we neglect the internal electric field $E(x)$ that may arise within the semiconductor. This approximation will be relaxed in question 1.4(d).

(a) Using Eq. (1.1) and the boundary conditions, express $\Delta p(x) = p(x) - p_0$ for any x and draw $\Delta p(x)$. You may introduce $L_{\mathrm{p}} = \sqrt{D_{\mathrm{p}} \tau_{\mathrm{p}}}$. What is the physical meaning of L_{p}?

(b) Assuming local quasi-neutrality, one can write $\Delta n(x) = n(x) - n_0 \approx \Delta p(x)$. Express $J_{\mathrm{p}}^{\mathrm{diff}}(x)$ and $J_{\mathrm{n}}^{\mathrm{diff}}(x)$.

(c) What must be the value of the total current density $J^{\mathrm{tot}}(x)$?

(d) Using this condition, express the electric field $E(x)$ as a function of L_{p}, D_{n}, D_{p}, σ_{n} and σ_{p}. Comment on this result.

We now consider that a constant excess of holes Δp_{l} is maintained for $x \leqslant 0$, whereas a similar deficit of holes Δp_{r} is maintained for $x \geqslant w$ ($w > 0$). In the following, we neglect the internal electric field.

(e) Solve again the continuity equation (1.1) and, using the boundary conditions, express $\Delta p(x)$ in the region $0 \leqslant x \leqslant w$. Give simplified expressions for $\Delta p(x)$ in the two limiting cases $w \gg L_{\mathrm{p}}$ (long device configuration) and $L_{\mathrm{p}} \gg w$ (short device configuration).

(f) Plot $\Delta p(x)$ in the two configurations introduced above.

(g) Determine the simplified expression of $J_{\mathrm{p}}^{\mathrm{diff}}(x)$ in the short and long device configuration, respectively. Comment on the differences between these two expressions.

2 Paramagnetism of degenerate and nondegenerate free electrons

We consider a three-dimensional (3d) gas of $N \gg 1$ noninteracting free electrons of mass m and spin $s = 1/2$ placed in a magnetic field B directed along the z axis. Each electron has a magnetic moment of modulus equal to the Bohr magneton μ_{B} with two possible orientations with respect to the applied magnetic field. We note T the temperature, $\beta = 1/k_{\mathrm{B}}T$, and μ the electron chemical potential.

(a) Show that the density of states $g(E)$ per unit volume for free electrons in a 3d system in the *absence* of a magnetic field as a function of the energy E is given by $g(E) = K\sqrt{E}$, where K is a constant to be determined.

(b) We apply a weak magnetic field such that $\mu_{\mathrm{B}}B \ll \mu$. Considering only the effect of the additional Zeeman term on the spin degree of freedom, give the new dispersion relation (with respect to the $B = 0$ case) and the densities of states $g_{\pm}(E)$ for spins parallel (+) and antiparallel (−) to the magnetic field B, respectively.

(c) In the degenerate case of $\beta\mu \gg 1$, indicate graphically the occupation as a function of energy $f(E)$ of electronic states at $T = 0\,\mathrm{K}$ and at $T > 0\,\mathrm{K}$.

(d) Calculate the number of electron spins per unit volume parallel (n_+) and antiparallel (n_-) to the magnetic field.

(e) Determine the magnetization M and the paramagnetic susceptibility χ of electrons at $T = 0\,\mathrm{K}$.

(f) We now consider the nondegenerate case of low electron density, where we can approximate the Fermi–Dirac distribution by $f(E) \approx \mathrm{e}^{-\beta(E-\mu)}$. Calculate M and χ as a function of the electron density and the temperature. Explain why the Curie law is obtained for the susceptibility of nondegenerate electrons.