

Formulaire

Notations

- un champ de vecteur est noté en gras : $\mathbf{v}$;
- un vecteur unitaire est noté avec un chapeau : $\hat{\mathbf{v}} = \mathbf{v}/|\mathbf{v}| = \mathbf{v}/v$;
- le produit vectoriel de deux vecteurs $\mathbf{A}$ et $\mathbf{B}$ est noté $\mathbf{A} \times \mathbf{B}$.

Systèmes de coordonnées

Coordonnées cartésiennes

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}; \quad d\tau = dx dy dz$$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}} \\ \nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\ \nabla \times \mathbf{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}} \\ \nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \end{aligned}$$

Coordonnées sphériques

$$\begin{aligned} x &= r \sin \theta \cos \varphi & \hat{\mathbf{x}} &= \sin \theta \cos \varphi \hat{\mathbf{r}} + \cos \theta \cos \varphi \hat{\boldsymbol{\theta}} - \sin \varphi \hat{\boldsymbol{\varphi}} \\ y &= r \sin \theta \sin \varphi & \hat{\mathbf{y}} &= \sin \theta \sin \varphi \hat{\mathbf{r}} + \cos \theta \sin \varphi \hat{\boldsymbol{\theta}} + \cos \varphi \hat{\boldsymbol{\varphi}} \\ z &= r \cos \theta & \hat{\mathbf{z}} &= \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} & \hat{\mathbf{r}} &= \sin \theta \cos \varphi \hat{\mathbf{x}} + \sin \theta \sin \varphi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ \theta &= \arctan(\sqrt{x^2 + y^2}/z) & \hat{\boldsymbol{\theta}} &= \cos \theta \cos \varphi \hat{\mathbf{x}} + \cos \theta \sin \varphi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}} \\ \varphi &= \arctan(y/x) & \hat{\boldsymbol{\varphi}} &= -\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}} \end{aligned}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\varphi \hat{\boldsymbol{\varphi}}; \quad d\tau = r^2 \sin \theta dr d\theta d\varphi$$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \hat{\boldsymbol{\varphi}} \\ \nabla \cdot \mathbf{v} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\varphi}{\partial \varphi} \\ \nabla \times \mathbf{v} &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\varphi) - \frac{\partial v_\theta}{\partial \varphi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r} (r v_\varphi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\varphi}} \\ \nabla^2 f &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2} \end{aligned}$$

Coordonnées polaires/cylindriques

$$x = r \sin \theta$$

$$y = r \cos \theta$$

$$z = z$$

$$\hat{\mathbf{x}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{y}} = \sin \theta \hat{\mathbf{r}} + \cos \theta \hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan(y/x)$$

$$z = z$$

$$\hat{\mathbf{r}} = \cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\theta}} = -\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}}$$

$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + dz \hat{\mathbf{z}}; \quad d\tau = r dr d\theta dz$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \mathbf{v} = \left[\frac{1}{r} \frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \right] \hat{\mathbf{r}} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[\frac{\partial}{\partial r} (rv_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Théorèmes fondamentaux d'analyse vectorielle

Théorème du gradient : $\int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{l} \cdot \nabla f = f(\mathbf{b}) - f(\mathbf{a})$

Théorème de la divergence : $\int_V d\tau (\nabla \cdot \mathbf{A}) = \oint_S d\mathbf{a} \cdot \mathbf{A} \quad (\text{Green-Ostrogradsky})$

Théorème du rotationnel : $\int_S d\mathbf{S} \cdot (\nabla \times \mathbf{A}) = \oint_{\mathcal{P}} d\mathbf{l} \cdot \mathbf{A} \quad (\text{Stokes})$

Constante universelle des gaz parfaits

$$R = 8.31 \text{ J.K}^{-1}.\text{mol}^{-1}$$