Université de Strasbourg
Master de Physique & Magistère, M1–S2

SOLID STATE PHYSICS
Academic year 2025–2026

Exam — Session 1

Duration: 2 h.

Documents, cell phones, computers, tablets, pocket calculators, etc., are not allowed.
The text contains 4 pages in total, and the 2 exercices are independent from one another.

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1 The *p*-*n* junction at equilibrium

1.1 The mass action law

We consider a three-dimensional homogeneous two-band, direct gap semiconductor, with an energy gap $E_\mathrm{g} = E_\mathrm{c} - E_\mathrm{v}$ between the conduction (c) and valence (v) band edges. The electron concentration in the conduction band and the hole concentration in the valence band at a temperature T are denoted $n_0(T)$ and $p_0(T)$, respectively. In this section, we study thermally assisted generation and recombination of electrons and holes in the semiconductor.

(a) We assume that the electron (or hole) generation rate writes $\mathcal{A}\exp(-E_\mathrm{g}/k_\mathrm{B}T)$, where k_B is the Boltzmann constant. Comment on this expression. What is the unit of the constant $\mathcal{A}$?

(b) We assume that the electron-hole pair recombination rate writes $\mathcal{B}\, n_0(T)\, p_0(T)$. Comment on this expression. What is the unit of the constant $\mathcal{B}$?

(c) Write the expression of $\dot{n}_0(T) \equiv \mathrm{d}n_0(T)/\mathrm{d}t$, *i.e.*, the kinetic equation for the electron concentration. In the steady state, give a simple expression of the product $n_0(T)\, p_0(T)$ in terms of the constants of the problem introduced above.

(d) Express the product $n_0(T)\, p_0(T)$ in the case of an intrinsic (*i.e.*, undoped) semiconductor and give the expression of the intrinsic charge carrier concentration $n_\mathrm{i}(T)$.

1.2 Doped semiconductors

For now, starting from the same intrinsic material, we separately consider a p-doped (hole doped) an a n-doped (electron doped) bulk semiconductor, where p-doping (resp. n-doping) arises from an impurity band, with acceptor (resp. donor concentration) denoted N_a (resp. N_d), situated at energies $\delta E_\mathrm{a} \ll E_\mathrm{g}$ (resp. $\delta E_\mathrm{d} \ll E_\mathrm{g}$) above the valence band maximum (resp. below the conduction band minimum).

(a) Take silicon (Si) as an example of a semiconducting material. How can one obtain p-doped Si and n-doped Si, respectively?

(b) In the following, we shall consider the *saturation regime*, *i.e.*, all acceptors and donors are ionized in the p-doped and n-doped region, respectively, but thermal generation of charge carriers across the semiconductor bandgap remains negligible. Define the temperature range that corresponds to the saturation regime.

(c) Express the charge neutrality condition in the n-doped and p-doped cases. Knowing that the *intrinsic* charge concentration at room temperature (300 K) is typically $n_\mathrm{i} \sim 10^{16}\,\mathrm{m}^{-3}$ and that $N_\mathrm{a,d} \sim 10^{22}\,\mathrm{m}^{-3}$, give a numerical estimate of n_0 in the p-doped region and of p_0 in the n-doped region, respectively, at room temperature.

(d) Define the “majority carriers” and “minority carriers” in the p-doped and n-doped regions, respectively.

(e) Draw the energy diagram of the p-doped semiconductor (top of the valence band, bottom of the conduction band, impurity band) and separately that of the n-doped semiconductor. Indicate qualitatively the position of the chemical potential μ_p in the p-doped case, and μ_n in the n-doped case, respectively.

1.3 Energy diagram and electrostatics of the *p-n* junction

The two energy diagrams drawn in Question 1.2(e) can be viewed as a *p*-*n* junction before contacting the *p*-doped and *n*-doped regions. We now explicitly consider the abrupt junction formed when the *p*-doped and *n*-doped semiconductors are contacted. For simplicity, we assume a one-dimensional problem along the $\hat{\mathbf{x}}$ axis, with the junction at $x = 0$, and translational invariance in the $(\hat{\mathbf{y}}, \hat{\mathbf{z}})$ plane.

(a) Draw the energy diagram of the *p*-*n* junction along the $\hat{\mathbf{x}}$ axis. Indicate the chemical potential of the junction and give the expression of the built-in electrostatic potential ϕ_i as a function of μ_p and μ_n.

(b) Explain how a *depletion region* (also known as *depletion layer*) forms near $x = 0$ when the *p*-*n* junction is formed.

(c) Express and represent graphically the charge carrier density $\rho(x)$ in and out of the depletion region and justify why this region is also called a *space-charge* region.
Hint: You may consider that the space-charge region spans from $x = -d_p$ to $x = 0$ on the p side of the junction and from $x = 0$ to $x = d_n$ on the n side of the junction (with $d_{p,n} > 0$).

(d) We introduce the electrostatic potential $\phi(x)$, which obeys Poisson's equation

$$\frac{\mathrm{d}^2\phi(x)}{\mathrm{d}x^2} + \frac{\rho(x)}{\epsilon} = 0, \tag{1.1}$$

where $\epsilon = \epsilon_0\epsilon_r$, with ϵ_0 the vacuum permittivity and ϵ_r the dielectric constant of the bulk semiconductor. We assume that $\phi(x)$ is constant outside of the depletion layer, *i.e.*, for $x \leqslant -d_p$ and $x \geqslant d_n$. The corresponding electric field $\mathcal{E}(x)$ is thus vanishing outside the deplation layer. Solve Eq. (1.1) and express $\phi(x)$ and $\mathcal{E}(x)$.

(e) Plot $\phi(x)$ and $\mathcal{E}(x)$.

(f) Considering the continuity of $\phi(x)$ and $\mathcal{E}(x)$ at $x = 0$, express d_p and d_n as a function of N_a, N_d, ϵ, ϕ_i and e, with e the elementary charge. Comment on these expressions. Show that $w = d_p + d_n$, the width of the depletion layer, writes

$$w = \sqrt{\frac{N_\mathrm{a} + N_\mathrm{d}}{N_\mathrm{a}N_\mathrm{d}}\frac{2\epsilon\phi_\mathrm{i}}{e}}.$$

(g) We give $e = 1.6 \times 10^{-19}\,\mathrm{C}$, $\phi_\mathrm{i} = 1\,\mathrm{V}$, $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{Fm^{-1}}$, $\epsilon_\mathrm{r} = 10$, and $N_\mathrm{a} = N_\mathrm{d} = 10^{22}\,\mathrm{m^{-3}}$. Give an estimate of w and give an order of magnitude for the built-in electric field $\mathcal{E}$ across the junction.

2 Antiferromagnetic simple cubic crystal

Let us consider a simple cubic lattice crystal of volume V, containing N atoms maintained at a temperature T. Each isolated atom has a spin angular momentum (in units of $\hbar$) $\mathbf{S}$. The magnetic moment of an atom, which comprise a single unpaired electron, is given by $\boldsymbol{\mu} = -g\mu_\mathrm{B}\mathbf{S}$, where $g = 2$ is the Landé factor and μ_B the Bohr magneton. An external magnetic field $\mathbf{B} = B\,\hat{z}$ is applied in the z direction corresponding to the [001] crystalline direction (and which defines the quantization axis).

In this exercice we consider an antiferromagnetic exchange interaction $J < 0$ between nearest neighbors on the lattice. The Hamiltonian of the system reads

$$H = -J\sum_{\langle i,j\rangle} S_i^z S_j^z + 2\mu_\mathrm{B}B\sum_{i=1}^{N} S_i^z, \tag{2.1}$$

where $S_i^z = \pm 1/2$ is the z component of the spin angular momentum (with $i = 1, \ldots, N$ the lattice site index) while $\langle i,j\rangle$ represents a summation over nearest neighbors.

2.1 Noninteracting case ($J = 0$)

We first consider the case where there is no exchange interaction between nearest neighbors, *i.e.*, $J = 0$. The Hamiltonian (2.1) can then be expressed as a sum over N independent terms as $H = \sum_{i=1}^{N} H_i$, with

$$H_i = 2\mu_\mathrm{B}BS_i^z.$$

(a) Calculate the (canonical) partition function Z of each atom.

(b) The free energy (per atom) is defined as $F = -k_\mathrm{B}T\ln Z$. Show that the mean value of the magnetic moment of each atom is given by

$$\langle\mu^z\rangle = -\frac{\partial F}{\partial B}.$$

(c) Show that the average magnetization in z direction $\langle M^z\rangle$, defined as the mean total magnetic moment per unit volume, is linked to $\langle\mu^z\rangle$ by the relationship

$$\langle M^z\rangle = n\,\langle\mu^z\rangle,$$

with $n = N/V$ the density of atoms in the crystal.

(d) Deduce from the preceding questions that the average magnetization can be expressed as

$$\langle M^z\rangle = M_\mathrm{s}\tanh(\beta\mu_\mathrm{B}B), \tag{2.2}$$

with $\beta = 1/k_\mathrm{B}T$ and where the saturation magnetization

$$M_\mathrm{s} = n\mu_\mathrm{B}. \tag{2.3}$$

(e) Sketch the magnetization (2.2) as a function of $\mu_\mathrm{B}B/k_\mathrm{B}T$ and comment on your result. Is the system paramagnetic, diamagnetic, ferromagnetic, or antiferromagnetic?

(f) The zero-field magnetic susceptibility is defined as

$$\chi = \mu_0\,\frac{\partial\langle M^z\rangle}{\partial B}\bigg|_{B=0},$$

where μ_0 is the vacuum magnetic permeability. Show that the susceptibility follows the Curie law $\chi = C/T$. Give the expression of the constant C in terms of μ_0, μ_B, k_B, and n.

2.2 Interacting case ($J < 0$)

We now consider the full Hamiltonian of Eq. (2.1) including an antiferromagnetic exchange interaction between nearest neighbors, *i.e.*, $J < 0$.

(a) Let us consider for this question that $T = 0$ and $B = 0$. Justify that the system splits into two sublattices A and B, such that the angular momenta take the value $+1/2$ or $-1/2$ depending on the sublattice to which they belong. These states are called *Néel states*. How many Néel states are there?

(b) Let us call $\langle M^A \rangle$ ($\langle M^B \rangle$) the average magnetization of the A (B) sublattice in the z direction (from now on, we omit for simplicity the z superscript). By writing down the energy E_i^A of one lattice site i belonging to the A sublattice and within the mean (molecular) field approximation due to Pierre Weiss, argue that the effective magnetic field seen by the spin S_i^A is given by

$$B_{\text{eff}}^A = B - \lambda \langle M^B \rangle,$$

where

$$\lambda = \frac{3|J|}{n\mu_{\text{B}}^2}.$$

Justify carefully the above expression of λ. Within the same approximation, what is the effective magnetic field B_{eff}^B seen by a spin S_i^B belonging to the B sublattice?

(c) Deduce from the preceding questions that $\langle M^A \rangle$ and $\langle M^B \rangle$ obey the set of coupled self-consistent equations

$$\langle M^A \rangle = \frac{M_{\text{s}}}{2} \tanh\big(\beta\mu_{\text{B}} \left[B - \lambda \langle M^B \rangle\right]\big), \tag{2.4a}$$

$$\langle M^B \rangle = \frac{M_{\text{s}}}{2} \tanh\big(\beta\mu_{\text{B}} \left[B - \lambda \langle M^A \rangle\right]\big), \tag{2.4b}$$

where M_{s} is defined in Eq. (2.3).

(d) Argue that for vanishing applied magnetic field ($B = 0$), $\langle M^A \rangle = -\langle M^B \rangle \equiv M$, so that the self-consistent equations (2.4) simplify to

$$m = \tanh\left(\frac{3\beta|J|}{2} m\right), \tag{2.5}$$

with $m = 2M/M_{\text{s}}$.

(e) Solve Eq. (2.5) graphically and discuss its solutions. In particular, show that there is a antiferromagnetic/paramagnetic phase transition at the Néel temperature T_{N}. Give the expression of T_{N} as a function of $|J|$. Sketch $|m|$ as a function of T/T_{N}. Where is the antiferromagnetic phase? The paramagnetic one?

(f) Still for $B = 0$, sketch the total magnetization $M_{\text{tot}} = \langle M^A \rangle + \langle M^B \rangle$ and the staggered magnetization $M_{\text{sta}} = \langle M^A \rangle - \langle M^B \rangle$ as a function of T.

(g) We now consider a finite applied magnetic field. Using Eqs. (2.4), determine the zero-field magnetic susceptibility for $T \gtrsim T_{\text{N}}$ and show that it can be expressed as

$$\chi = \frac{C}{T + T_{\text{N}}},$$

where C is the constant determined at Question 2.1(g). Sketch χ as a function of T.

(h) If the exchange interaction were ferromagnetic, what would be the behavior of the magnetic susceptibility in the vicinity of the critical temperature (no detailed justification needed)?