Mesoscopic Systems with Disorder and Interaction

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Abstract

Mesoscopic systems with disorder and interaction can be realizations of interesting quantum many-body problems. Since systematic experimental and theoretical studies of a great variety of mesoscopic systems are possible, they play an important role in fundamental research. At the same time, mesoscopic systems are also discussed in view of possible applications in nanoelectronic devices for metrology, and quantum as well as classical computers. For both, the fundamental research and the applications, the electronic properties of the samples are most promising and therefore particularly interesting.

Many features of different types of mesoscopic systems are well understood from effective one-particle theories. However, some experimentally observed phenomena like the value of persistent currents in diffusive rings or the existence of a metallic phase in disordered two-dimensional samples cannot be obtained within models neglecting either the interactions between the electrons or the disordered potential. A more realistic description of such systems requires to take into account both, disorder and interaction, on equal footing and represents a challenge to theoretical physics.

A few years ago, a new approach to this problem was proposed by Shepelyansky. It consists of considering a small number of particles, but allowing for strong disorder and strong interaction between them. Even the simplest case of only two particles in a one-dimensional disordered chain leads to novel and unexpected results. Among the most prominent ones, it is the increase of the pair localization length due to a moderate interaction, between the two strongly localized limits of vanishing and very large interaction strength. As in the one-particle problem in disordered samples, the many-body spectral statistics is closely related to the localization properties of the two-electron wave-functions, establishing a fruitful connection to the field of quantum chaos.

The extension of this approach to a larger number of particles yields informations about the influence of the interaction on the conductance of disordered systems. Recent results indicate that moderate interactions can indeed delocalize the many-body ground state in disordered one- and two-dimensional samples, and support the following scenario for the experimentally observed metal-insulator transition as a function of the carrier density in two-dimensional samples: At high density, the interactions can be neglected and one has an Anderson insulator. At low density, the interactions impose a Wigner crystal which is pinned by the disorder. Between these two insulating limits, moderate interactions might lead to non-trivial electronic correlations and allow for metallic behavior.

Contents

1 Introduction

Towards the end of the XX$^{\text{th}}$ century, considerable advances in the controlled fabrication of sub-micron solid state structures, as well as the common availability of low temperature facilities, have allowed for a systematic investigation of artificially made structures whose electronic properties are modified or even dominated by quantum interference effects. This makes it possible to perform experiments in the mesoscopic regime which directly probe quantum properties of phase coherent many-body systems. While a variety of phenomena can be understood within a (weakly interacting) quasi-particle approach [54], the always present strong Coulomb interaction between electrons can have a dramatic influence on the transport properties when the electrons are confined in low-dimensional systems. A prominent example is the Coulomb blockade effect and the resulting conductance oscillations in the transport through quantum dots [62]. The confinement is expected to render electronic correlations more important than in the bulk. Going to lower dimensions and/or very dilute limits results in a poorer screening of the electron-electron interaction, enhancing the role of Coulomb repulsions. When the disorder is strong, quantum interference leads to Anderson localization [11] of the electron wave functions. The resulting suppression of the mobility of the charge carriers becomes also detrimental to screening, thereby further amplifying the role of interactions. On can therefore expect important effects of the electron-electron interaction and possibly a breakdown of the Fermi-liquid quasi-particle approaches in confined and/or disordered systems.

In the following section, we introduce two important experimental findings which differ considerably from the result of effective one-particle theories. These examples underline the necessity to develop theories which take into account both, strong interactions and strong disorder.

1.1 Experimental motivation

1.1.1 Persistent currents

When phase-coherent rings are pierced by a magnetic flux, a finite current typically flows around the ring in thermodynamic equilibrium. The measured value of this persistent current in diffusive rings [73, 26] is much larger (at least an order of magnitude) than the theoretical prediction for non-interacting electrons in disordered rings [95, 118, 7]. While the electron-electron interaction seems to play an essential role, the disorder in the sample is also important: Interactions cannot affect the persistent current in

clean rotationally invariant one-dimensional rings [99, 85, 84], and the non-interacting result is consistent with the experimental one for a clean semiconductor ring in the ballistic regime [77].

This has generated a large theoretical activity, dealing with the combined effect of interactions and disorder on the enhancement of persistent currents in mesoscopic rings (for an overview see e.g. [54, 78, 33] and references therein). Even though different theoretical approaches suggest an increase of the persistent current in disordered samples due to repulsive Coulomb interactions, a quantitative understanding of the experiments is still lacking.

1.1.2 Two-dimensional metal

The recent observation of a metallic behavior in two-dimensional gases of electrons (Si-MOSFET [66, 67, 68]) or holes (GaAs heterostructures [50] and SiGe quantum wells [70, 29]) is at odds with the conclusions of non-interacting theories that predict an insulating behavior for any value of the disorder strength [3]. For a recent review on the metal-insulator transition in two-dimensional electron and hole gases see [4]. The systems are insulating at very low charge carrier density, and become metallic when the carrier density is increased. This transition might be related to the melting of a pinned Wigner crystal [14]. The re-entrance into the insulating regime at very high carrier density was observed in [50]. This may indicate that screening becomes efficient enough to suppress the electronic correlation effects, leading to the Anderson insulator predicted without interactions.

1.2 Anderson localization

The theoretical expectation of insulating behavior in two-dimensional systems is based on the localization of electronic wave functions in disordered samples. Anderson showed [11] that quantum interference can lead to exponentially localized electronic wave functions (for a review on Anderson localization see [65]).

One of the simplest models describing electrons in disordered systems is the Anderson model of non-interacting particles on a lattice (all lengths are measured in units of the lattice constant a)

$$H_{\mathrm{A}} = -t \sum_{\sigma} \sum_{<i,j>} \left(c^{+}_{i,\sigma} c_{j,\sigma} + c^{+}_{j,\sigma} c_{i,\sigma} \right) + \sum_{\sigma} \sum_{i} v_i n_{i,\sigma} \qquad (1)$$

with hopping between next neighbor sites $< i, j >$. The operators $c_{i,\sigma}$ ($c^{+}_{i,\sigma}$) destroy (create) a particle with spin $\sigma = \{\uparrow, \downarrow\}$ on site i and $n_{i,\sigma} = c^{+}_{i,\sigma} c_{i,\sigma}$ is the occupation number operator. The on-site random energies v_i are drawn

independently from a box distribution in the interval $[-W/2, +W/2]$. The strength of the disorder W is measured in units of the kinetic energy scale ($t = 1$). The presence of the disorder leads to a finite mean free path l of the particles. The asymptotic behavior of the wave-functions far away from the center $\vec{r}_0$ of their localization domain is roughly given by

$$|\psi_i| \propto \exp(-|\vec{r}_i - \vec{r}_0|/L_1) \tag{2}$$

and is characterized by the one-particle localization length L_1 (in units of the lattice spacing).

Within the Anderson model, and in one dimension, the one-particle localization length in the band center at not too strong disorder is given by [112]

$$L_1 \approx 96/W^2 \,. \tag{3}$$

This means that the electronic wave-functions are always localized in one dimension, provided the disorder W does not vanish.

1.2.1 Scaling theory of localization

In order to obtain the behavior of the wave-functions in two- and three-dimensional systems, the dependence of the conductance g on the linear system size L has been investigated for d-dimensional hypercubes of volume L^d [3].

One assumes that the scaling of the system size $L \to bL$ leads to a new conductance

$$g(bL) = F_d(b, g(L)) \tag{4}$$

which depends only on the scaling factor b and the original conductance, but not on individual details like disorder strength or Fermi energy. This means that the function

$$\beta(g) = \frac{\mathrm{d} \ln g}{\mathrm{d} \ln L} \,, \tag{5}$$

determining the L-dependence of the conductance, depends only on g itself.

At strong disorder, the localization of the electronic states implies that the conductance scales like

$$g \propto \exp(-2L/L_1) \tag{6}$$

with a dimension-dependent prefactor, provided the length of the system L exceeds the localization length L_1. This leads to

$$\beta(g) \sim \ln g \,. \tag{7}$$

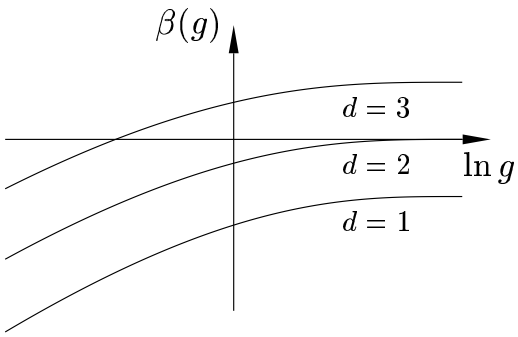

Figure 1: Scheme of the scaling curves $\beta(g)$ in one, two and three dimensions.

At very weak disorder, in the diffusive regime, the conductance

$$g \propto L^{d-2} \tag{8}$$

is given by the conductivity multiplied by the cross section L^{d-1} and divided by the length of the sample, yielding

$$\beta(g) = d - 2\,. \tag{9}$$

The lowest order quantum interference corrections (so-called weak localization) to β are negative, and proportional to $1/g$.

For the rough overall behavior of the function $\beta(g)$, one smoothly and monotonously interpolates between the previous limits and obtains the curves shown in Fig. 1. The scaling assumptions and the main results were confirmed using a systematic self-consistent treatment of the one-particle diffusion [117].

The sign of β determines how g changes with L. For one- and two-dimensional systems, β is always negative and therefore the conductance scales to zero in the thermodynamic limit of infinite system size, implying insulating behavior. Only in three (and more) dimensions, a critical conductance exists above which the conductance increases with system size, indicating metallic behavior.

From this one concludes that non-interacting one- and two-dimensional systems cannot be metallic. The perturbative inclusion of repulsive electron-electron interactions leads to corrections which further decrease the conductance [6]. Together with the well-known Mott-Hubbard transition where strong repulsive interactions lead to insulating behavior even in the clean Hubbard model, this lead to the belief that repulsive interactions have tendency to reduce the conductance and to strengthen localization in one and

two dimensions. On these grounds, all disordered one-and two-dimensional systems with or without repulsive electron-electron interactions were expected to be insulators and the recent discovery of a metallic phase in two-dimensional systems (see section 1.1.2) came as a big surprise.

1.2.2 Level statistics

The localization of the electronic one-particle wave-functions is closely related to the statistics of the corresponding energy levels [83]. Investigating the spectral statistics is an efficient tool to detect the metal-insulator transition in three-dimensional systems [104] and the cross-over between the localized and the diffusive regime in finite size samples. These effects on the spectral statistics allow to make a connection to the field of quantum chaos [48].

The most important measures of the spectral fluctuations include the probability distribution of the spacing between adjacent energy levels $P(s)$ and the variance $\Sigma_2(E)$ of the number of levels found in an energy interval of width E, where s and E are usually measured in units of the mean level spacing Δ.

When the wave-functions are localized in a small fraction of the sample volume ($L \gg L_1$), energetically neighboring states see different parts of the random potential and are completely uncorrelated. This leads to Poissonian level statistics and

$$P(s) = \exp(-s) \qquad \Sigma_2(E) = E\,. \tag{10}$$

In the diffusive regime, when $L_1 \gg L \gg l$ with the elastic mean free path l, the wave-functions are extended over the whole sample and the energy levels become correlated. The spectral statistics approaches the universal Wigner-Dyson statistics found for random matrices [79] within the Gaussian Orthogonal Ensemble (GOE).

$$P(s) \approx \frac{\pi}{2} s \exp(-\pi s^2/4) \qquad \Sigma_2(E) \propto \ln E \quad \text{for } E \gg 1\,. \tag{11}$$

This indicates that a diffusive mesoscopic system can be considered as chaotic in the sense of quantum chaos [48] and a random-matrix description of the spectral fluctuations [47] is possible.

The metal-insulator transition in three or more dimensions is detected from the scaling of the level statistics with increasing system size [104]. On the insulating side, the statistics approaches Poisson, while it tends to Wigner-Dyson on the metallic side. Exactly at the transition, the statistics is independent of the system size [104, 69, 128, 24] and close to the so-called

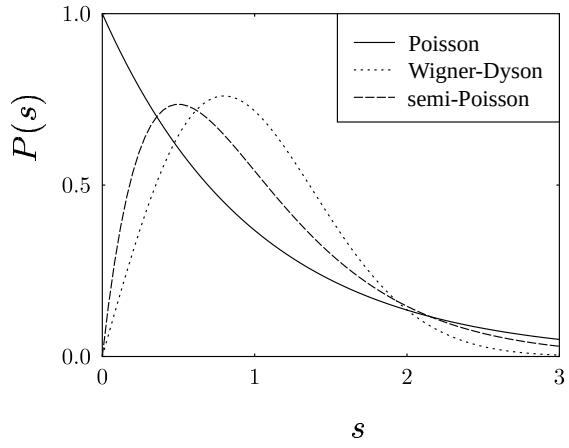

Figure 2: The level spacing probability distribution for Poisson, Wigner-Dyson, and semi-Poisson statistics.

semi-Poisson statistics

$$P(s) \approx 4s \exp(-2s) \qquad \Sigma_2(E) \propto \chi E \quad \text{for } E \gg 1\,, \tag{12}$$

showing level repulsion at small energy differences and weak correlations ($\chi < 1$) at large energy scales. This critical statistics characterizes weak chaos and appears in several other quantum systems like Kepler billiards [9], certain triangular billiards [20], and also in the Harper model [35]. The different curves for $P(s)$ are shown in Fig. 2.

In the realistic case of diffusive metals with a finite conductance g, the energy level correlations follow the Wigner-Dyson statistics (11) up to an energy scale [10] which is given by the Thouless energy E_c. The Thouless energy is directly proportional to the conductance [34, 110], such that the energy level statistics does not only allow for the distinction between the different transport regimes, but also to quantitatively measure the conductance.

However, these relations between level statistics and transport properties in disordered systems are well-established only for the case of non-interacting particles and the statistics of the one-particle energy levels. In the presence of interactions, one has to consider the many-body energy levels and it is *a priori* not at all clear whether a similar relation between the many-body level statistics and transport properties also holds for interacting systems in the presence of disorder and how the interactions will modify the one-particle picture presented above.

2 Two interacting particles in a disordered chain

In 1994, Shepelyansky [100] studied the simplest situation exhibiting the interplay between disorder and interaction: Two particles in a one-dimensional disordered chain with a local Hubbard-like interaction of strength U. The corresponding Hamiltonian

$$H = H_{\mathrm{A}} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} \tag{13}$$

is given by the Anderson Hamiltonian of equation (1), containing the disordered potential and the kinetic energy terms, and an additional term accounting for the interaction. This Hamiltonian is restricted to two fermionic particles of different spin with an antisymmetric spin part of the wave-function, such that only symmetric real space wave-functions will be considered in the following. Antisymmetric real-space wave-functions are of no interest since the interaction term is irrelevant in this case.

One works in the basis of the non-interacting two-particle states $|\alpha_1, \alpha_2\rangle$ with the real space representation

$$\langle i_1, i_2 | \alpha_1, \alpha_2 \rangle = \chi_{\alpha_1,\alpha_2}(i_1, i_2) = \frac{1}{\sqrt{2}} \left(\phi_{\alpha_1}(i_1)\phi_{\alpha_2}(i_2) + \phi_{\alpha_1}(i_2)\phi_{\alpha_2}(i_1) \right) , \tag{14}$$

constructed from the one-particle solutions $\langle i|\alpha\rangle = \phi_\alpha(i)$ of H_{A} corresponding to the one-particle energies ϵ_α. Then, the Schrödinger equation for the components $\psi(\alpha_1, \alpha_2)$ of the interacting two-particle wave-functions $|\psi\rangle = \sum_{\alpha_1,\alpha_2} \psi(\alpha_1, \alpha_2)|\alpha_1, \alpha_2\rangle$ reads

$$(\epsilon_{\alpha_1} + \epsilon_{\alpha_2})\psi(\alpha_1, \alpha_2) + 2U \sum_{\alpha_3,\alpha_4} Q_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} \psi(\alpha_3, \alpha_4) = E\psi(\alpha_1, \alpha_2) \, . \tag{15}$$

The matrix Q coupling different non-interacting eigenstates is given by

$$Q_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} = \sum_i \phi^+_{\alpha_1}(i)\phi^+_{\alpha_2}(i)\phi_{\alpha_3}(i)\phi_{\alpha_4}(i) \, . \tag{16}$$

Ordering the one-particle wave-functions ϕ_α according to the position $m(\alpha)$ of their localization domain on the chain, the matrix elements will be exponentially suppressed except for the cases when $m(\alpha_1)$, $m(\alpha_2)$, $m(\alpha_3)$, and $m(\alpha_4)$ all lie within a one particle localization length L_1 of each other. In the latter case, roughly approximating the localized one-particle wave-functions by $(1/\sqrt{L_1}) \exp(\mathrm{i}\theta_i)$ with random phases θ_i inside their localization domain and assuming zero outside, Shepelyansky estimated the coupling terms to be of the order

$$Q_{\mathrm{typ}} \approx 1/L_1^{3/2}. \tag{17}$$

2.1 Delocalization by interactions

Dropping the non-interacting two-particle states in which the particles are localized far apart ($|m(\alpha_1) - m(\alpha_2)| > L_1$), Shepelyansky mapped the remaining problem onto a band random matrix with a strong diagonal. From a numerical investigation of this random matrix at zero energy (in the band center) [100], he found that the two-particle wave-functions are localized along the center-of-mass direction ($m(\alpha_1) \approx m(\alpha_2)$) on a length scale

$$L_2 \approx L_1^2 \frac{U^2}{32t^2}\,, \tag{18}$$

provided $L_1 \gg 1$ and the interaction U is of the order of the kinetic energy scale t. This result has several remarkable features. First of all, L_2 is proportional to the square of L_1 and can therefore become much larger than the one-particle localization length. This means that, contrary to earlier expectations, the interaction leads to a delocalization[1] of the two-particle wave-function along the center-of-mass coordinate. The two particles seem to stay close to each other because of the one-particle localization and diffuse together due to the interaction. Let us mention that a similar result had been obtained earlier by Dorokhov [32], but using an attractive harmonic interaction between the two particles.

Secondly, the delocalization is proportional to the square of the interaction strength and therefore does not depend on the sign of the interaction. This feature is surprising at first glance, since attractive and repulsive interactions are usually expected to have very different consequences on electronic transport. It can however be traced back to a symmetry of the ensemble of disordered Hamiltonians with respect to the transformation ($E \to -E$; $U \to -U$). In the band center $E = 0$, a change of the sign of the interaction U must therefore leave the result unaffected.

Let us mention that a similar approach has been used previously, studying the effect of the electron-electron interactions by considering the resulting matrix elements in the non-interacting basis [61]. Very few hopping terms, induced by a Coulomb interaction, were taken into account to calculate the interaction dependence of the magneto-conductance, the electronic specific heat and the static spin susceptibility of disordered insulators, in the limit of small one-particle localization length L_1. Here, we are interested in the interaction effects at smaller disorder, when the interaction-induced two-particle hopping plays a much larger role.

[1] By delocalization, we mean an increase of the localization length, but not necessarily the emergence of extended states.

Imry [52] proposed a generalized Thouless block scaling [111] picture for the same problem of two interacting particles, but in disordered systems of arbitrary dimension d. Considering blocks of linear size L_1, the spacing of two-particle levels inside a block is of the order of

$$\Delta_2 \sim B/L_1^{2d} \tag{19}$$

with the bandwidth B being of the order of t for not too strong disorder and interaction. Neglecting the coupling between two-particle states in neighboring blocks through the one-particle hopping because of the one-particle localization, one can concentrate on the coupling due to the interaction. The typical coupling matrix element is (following Shepelyansky's assumptions leading to (17)) estimated to be of the order $U_{\rm typ} \approx U/L_1^{3d/2}$. Then, Fermi's golden rule gives the decay rate

$$\Gamma_2 \sim \frac{U_{\rm typ}^2}{\Delta_2} \tag{20}$$

of a two-particle state in a block due to the coupling to the neighboring blocks, and the dimensionless two-particle conductance is obtained to be

$$g_2 \sim \frac{\Gamma_2}{\Delta_2} \sim L_1^d \frac{U^2}{B^2}\,. \tag{21}$$

Using quasi-one dimensional scaling $g_2 \propto 1/L$ in the regime $L_1 < L < L_2$, the two-particle localization length L_2 is obtained from the length L where $g_2(L) = 1$. This yields

$$L_2 \propto L_1^{d+1} \frac{U^2}{B^2}\,, \tag{22}$$

in agreement with Shepelyanskys result (18) for $d = 1$, but suggests also, that the delocalization effect due to the interaction might become even stronger in higher dimensions [52, 53]. In this derivation of the delocalization, it becomes transparent that the high density of two-particle levels $1/\Delta_2$, much larger than the density of one-particle levels, plays an important role for the effect.

2.1.1 Confirmation of the delocalization effect

Since the delocalization due to repulsive interactions was a surprise, and because it was obtained using rough approximations, several numerical investigations using different approaches and methods have been performed in order to check the result.

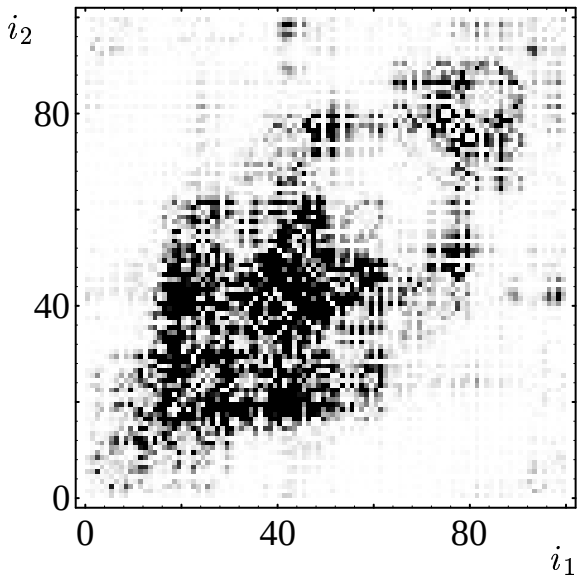

Figure 3: Grey scale plot of the absolute square $\mid \psi_{i_1,i_2} \mid^2$ of a numerically calculated interacting two-particle wave-function as a function of the one-particle coordinates i_1 and i_2, in a disordered ($W = 3$) one-dimensional chain of length $L = 100$. Dark regions correspond to high values. One observes the delocalization of the state along the diagonal $i_1 = i_2$. The figure is taken from [A1].

Shepelyansky and Borgonovi used the analogy [37] of the Anderson Hamiltonian to the kicked rotator and found evidence for the delocalization effect by interactions from numerical simulations of kicked rotators [100, 21, 22].

A rather direct confirmation was provided by a numerical diagonalization of the Hamiltonian for two interacting particles in one-dimensional rings, pierced by a magnetic flux [A1]. The disorder was chosen such that the one-particle localization length L_1 is much smaller than the circumference L of the ring. Then, one-particle propagation around the ring is suppressed exponentially. If, as predicted, the localization length L_2 for coherent two-particle propagation is considerably larger than L_1, two-particle processes are expected to dominate the magnetic response. In order to have $L_2 \gg L_1$, one needs large $L_1 \gg 1$. On the other hand, $L \gg L_1$ is needed for a suppression of the one-particle processes, posing a problem for numerical approaches since the dimension of the two-particle Hilbert space increases strongly with L.

Therefore, the Lanczos algorithm was used to diagonalize numerically [A1] the two-particle Hamiltonian, for large ring sizes of up to $L = 150$, at

$L_1 \approx 11$ (in these conditions an enhancement of L_2 by about a factor of two is expected). The energies of some two-particle eigenstates close to the band center were found to exhibit $h/2e$-periodic oscillations with the flux, showing that the particles propagate indeed together around the ring. This view is confirmed by an analysis of the decrease of the amplitude of these $h/2e$-periodic oscillations with the size of the rings, which is governed by the scale of the two-particle localization length L_2. In addition, a calculation of the corresponding wave-functions [A1] shows that their shape is extended along the center-of-mass direction. An example of such a wave-function is presented in Figure 3. A subsequent systematic study of the shape of such wave-functions [36] confirmed the existence of different types of two-particle states. In many states, the two particles are localized far apart and their energies are not sensitive to the interaction, but some states with overlapping one-particle wave-functions are affected and delocalized by interactions.

A related study concerning the real-time development of two-particle wave-packets [49, 31] showed that their spread along the center-of-mass direction increases with the interaction strength. This effect is also predicted for particle-hole pairs in semiconductors [25] and might be accessible to direct experimental observation.

The two-particle localization length L_2 was also calculated directly, using numerical transfer matrix methods similar to the ones used to investigate one-particle localization [90, 89, 76]. Even though the problem of two particles in one dimension is formally equivalent to a one-particle problem in two dimensions (each of the dimensions corresponding to one particle coordinate) with correlated disorder, some additional difficulties arise. First of all, the two particles are in the same chain, and the two-dimensional configuration space should be of square shape. Thus, the usual quasi-one dimensional transfer matrix iterations seem to be problematic since in one-particle studies of two-dimensional samples, they are usually continued in a given direction until the procedure converges. Second, the expected delocalization effect is not isotropic in the two-dimensional configuration space and the direction along which the transfer iterations are processed should matter.

Two different methods were used in [A2] to overcome these problems. The first consists of averaging the results for the transfer along a one-particle coordinate of many square-shape samples of a given size. In the second approach, the system is truncated at large relative coordinate (much larger than L_1) and the quasi-one dimensional transfer-matrix iterations are performed along the center-of-mass coordinate. From both methods, a delocalization effect of the interaction was found. However, the obtained result $L_2 \propto L_1^\alpha$ with $\alpha \approx 1.65$ was slightly different from the expected value of $\alpha = 2$. Moreover, the interaction dependence could not be extracted.

An independent method was proposed by von Oppen and coworkers [121] who extracted the two-particle localization length *via*

$$\frac{1}{L_2} = -\lim_{|i-j|\to\infty} \frac{1}{|i-j|} \ln |\langle i,i|G|j,j\rangle| \tag{23}$$

from matrix elements of the two-particle Green function G corresponding to the propagation along the center-of-mass axis in finite (large) size squares. They found $L_2 \approx L_1/2 + A|U|L_1^2$ for the present problem, as well as for antisymmetric real-space wave-functions with nearest-neighbor interactions (with a slightly different constant A). The anisotropy in the configuration space was confirmed by calculating different localization lengths for different propagation directions [106]. In addition, using this method, it was also found that away from the band center the strength of the delocalization effect depends on the sign of the interaction [120] and becomes very small when low-energy quasi-particle pairs above a filled Fermi sea are considered [119].

Using a decimation method [71, 92] for these Green function matrix elements, more support for the enhancement of the two-particle localization length due to interactions was obtained.

In parallel, analytical investigations mapping the problem to an effective σ model [39] could confirm the diffusive character of pairs in the range $L_1 < L < L_2$.

Of course, the delocalization due to the local interaction holds only while the interaction strength U is not much larger than the kinetic energy scale t. When $U \gg t$, the two-particle band is split and the enhancement of the localization length is suppressed [36, 91].

However, the existence of the delocalization effect was questioned [93] on the basis of numerical results for finite size squares, put together to a strip, which seemed to indicate that L_2 shrinks to L_1 in the extrapolation to infinite system size. It was argued that this was due to a misinterpretation of the data [A3] (see also [94]). A detailed discussion of the different methods and their results can be found in [40].

2.2 Level statistics and localization

The connection of the delocalization effect to the properties of random band matrices with a strong diagonal lead to further investigations of such random matrix ensembles. Analytical work using the supersymmetry technique [42, 38] and numerical work [55] showed that the local spectral density follows a Breit-Wigner form whose width Γ increases with the typical size of the off-diagonal matrix elements. On the other hand, the structure of the random

matrix ensemble corresponding to two interacting particles was found to deviate [127] from the supposed random band matrix.

Nevertheless, a Breit-Wigner local spectral density was also found for microscopic models of two interacting particles [A4, A5, 58, A6], where Γ increases with the interaction strength U (which determines the off-diagonal matrix elements in the Hamiltonian). The width Γ of the peak in the local spectral density characterizes the mixing of the (non-interacting two-particle) basis states by the off-diagonal interaction terms of the Hamiltonian. Even though this means that Γ depends on the choice of the basis, the latter is, at least at moderate interaction strength, not arbitrary. The structure of the two-particle Hamiltonian and the corresponding random matrices is dominated by strongly fluctuating diagonal entries (the sum of two one-particle energies $\epsilon_{\alpha_1} + \epsilon_{\alpha_2}$) and weakly fluctuating off-diagonal entries (of the order of $U_{\rm typ}$). This breaks the symmetry and installs the basis of the non-interacting states as a preferential basis of this problem. On energy scales of the order of the two-particle level spacing $\Delta_2 \sim B/L_1^{2d}$, the energy level statistics of the corresponding non-interacting two-particle levels is close to the Poisson-statistics [A5]. The correlations of the one-particle levels lead to correlations in the two-particle spectrum only at the energy scale of the one-particle level spacing Δ_1 which is much larger than Δ_2. The off-diagonal matrix elements lead to low-energy correlations in the energy spectrum, which is of course independent of the basis choice and the interacting two-particle spectrum exhibits universal random-matrix like correlations up to an interaction-dependent energy scale E_2, corresponding to a two-particle Thouless-energy [A4].

These correlations can be related [A4] to the width Γ of the eigenfunctions in the preferential basis. Since E_2 and Γ describe the mixing of different non-interacting wave-functions by the interaction, they are also related [A4, 58] to the two-particle delocalization described above, in the case of one-particle wave-functions which are localized in real space. This establishes a relation between the two-body level statistics and the localization, similar to the known relation for the one-particle problem.

If one increases the interaction strength U to values much larger than t and W, the states with both particles on the same site decouple from the rest of the spectrum [36, 91, A6] and the remaining states become effectively uncorrelated with energies approaching the non-interacting ones $\epsilon_{\alpha_1} + \epsilon_{\alpha_2}$ for $\alpha_1 \neq \alpha_2$. Then, the level statistics of two interacting particles in a one-dimensional chain becomes uncorrelated and the structure of the wave-functions can be expressed in terms of products of one-particle wave-functions as for $U = 0$. The mixing of the wave-functions by the interaction terms at small U can be related [A6] to the effect of the kinetic energy t^2/U in terms

of the interaction energy by a duality relation. In the intermediate regime $U \approx t$, and for a disorder W chosen such that the localization length L_1 equals the system size, the mixing of the basis states and the correlation of the energy levels is maximal. These correlations are found [A6] to be very close to the critical semi-Poisson statistics. At the same time, the wave-functions show multi-fractal features [A6], very much like one-particle wave-functions at the Anderson transition. The multi-fractal properties [123] of the interaction matrix elements (16) might be related to the multi-fractal properties of the interacting two-particle wave-functions.

While the correlations in the two-particle spectrum generated by the interactions are a signature of delocalization in the localized regime, the situation is very different at weak disorder strength. In the ballistic as well as in the diffusive regime, the one-electron wave-functions are extended. The correlation of the levels and the mixing of the non-interacting states caused by the interactions does not necessarily mean that the carriers become more mobile. On the contrary, it seems that the mixing of the states might rather be an indication for the presence of electron-electron scattering events which reduce the transport in this case. In order to clarify this question, the two-particle level curvatures were investigated [5] in a ring geometry as a function of the magnetic flux threading the ring. The one-particle curvatures are proportional to the conductance in the one-particle problem [111, 110], and the flux-dependence of the many-body ground state can even be related [64] to the Kubo conductance, such that the curvature statistics is expected to yield more direct informations about the transport properties than the energy level statistics.

In the two-particle problem it was predicted [5] from heuristic arguments that the mixing of the non-interacting basis states by the interaction should lead to an increase of the curvatures in the localized regime, consistent with the delocalization effect, but reduce them in the diffusive regime, indicating that interactions suppress transport in this case. A numerical investigation [A7, 126] confirmed this result and showed that the interaction-induced change in curvature scales with the non-interacting curvature. Whether the transport is enhanced or suppressed by the interactions therefore depends only on the non-interacting curvature, the latter being proportional to the non-interacting conductance of the system.

This conclusion is directly related to the one drawn from studies [49, 31, 25] of the real-time development of two-particle wave-packets. When interactions are present, the spread of these packets is slower before they reach the size L_1 (corresponding to the diffusive regime), but they continue to spread beyond that limit due to the delocalization in the localized regime. All these results on localization, level statistics, level curvatures and the real-

time development of wave-packets are closely related, also to the ground state properties of larger numbers of correlated fermions in disordered systems. For an overview, see [A8].

Let us note that not only a delocalization but even a transition to extended states along the center-of-mass direction was found for two particles in two dimensions [86]. The transition to Wigner-Dyson energy level statistics proposed for the same system [30] is however based on numerical calculations for relatively small systems and might be expected to disappear, for the case of short range interactions, in larger systems when uncorrelated states with particles localized far apart dominate the spectrum. In the case of long-range Coulomb interactions, a similarity of the problem of two interacting particles in two dimensions with the Anderson transition in three dimensions indicates the possibility of a transition to delocalized states [102].

3 Few-particle systems and the life-time of quasi-particles

Experimentally observable low-temperature quantum transport properties depend on the structure of the many-body energies and wave-functions. In the non-interacting case, the latter can be expressed as products of one-particle wave-functions and the transport properties can be traced back to the non-interacting spectrum. When interactions lead to strongly correlated many-body wave-functions, this is no longer possible. Neither is it to express the wave-functions in terms of the interacting two-particle states, and the properties of the two-particle spectrum cannot be related directly to the many-body transport properties. Having understood many features of the two-particle problem, the way to proceed consists rather in the increase of the number of particles, trying to extrapolate to the full many-body state. Thus, the spectral fluctuations which had proven to be a useful tool to characterize interacting two-particle systems, were investigated also in interacting few- and many-body systems.

3.1 Few-particle spectral statistics

Already for the case of three particles, an important new feature appears in the problem. Because the interaction has a two-body character, it does not couple all of the non-interacting basis states. Thus, the Hamiltonian matrix, in the basis of the non-interacting eigenstates (Slater determinants), is a sparse matrix, in contrast to the case of two particles. This sparseness becomes more and more pronounced when the number N of particles is increased. While the delocalization effect is expected to be stronger than for two particles [103], this has important qualitative consequences for the level statistics and the structure of the wave-functions in the non-interacting basis. In [A9], the dependence of the level statistics on the strength U of a local interaction is discussed using analytical arguments. The results are confirmed numerically for the case of three particles. The investigated quantity is the Thouless number g_N, given by the number of consecutive N-body levels which exhibit the universal Wigner-Dyson spectral rigidity. In the limit of a low density of particles, two interaction scales U_{c1} and U_{c2} are found. Below U_{c1}, the interaction mixes only weakly the Slater determinants which are very close in energy, and $g_N < 1$. Adjacent levels, separated by an energy which is of the order of the N-body level spacing Δ_N, are typically not directly coupled by the interaction. The effective coupling via other states can then be found perturbatively, leading to power laws $g_N \propto |U|^P$, with the exponent

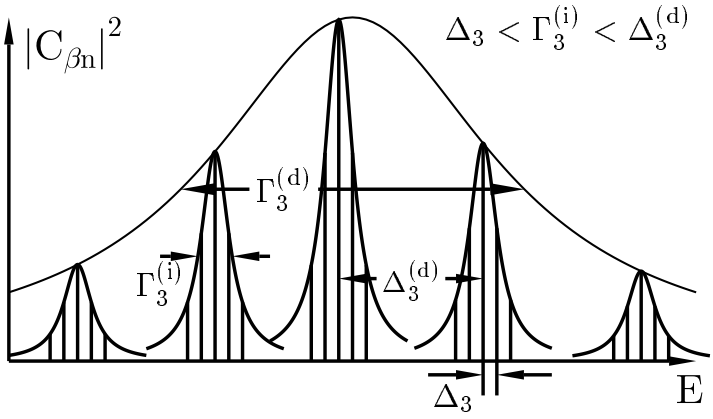

Figure 4: Sketch of the typical structure of an interacting N-body eigenfunction $|\psi_\beta\rangle$, for the intermediate interaction regime $U_{c1} < U < U_{c2}$, in the basis of the non-interacting eigenstates $|A_n\rangle$. The structure is characterized by the dependence of the overlap $C_{\beta n} = \langle\psi_\beta|A_n\rangle$ on the energy E of the basis state, for the example $N = 3$. The quantities $\Gamma_3^{(d/i)}$ and $\Delta_3^{(d)}$ are explained in the text. The figure is taken from [A9].

P depending on the particle number. One finds $P = N/2$ for N even and $P = (N + 1)/2$ for N odd. The scale U_{c1} characterizes the threshold above which nearest neighbor levels in energy start to be correlated. Even though the N-particle density of states is very large, this threshold is connected [103, A9] to the two-particle level spacing Δ_2, since the latter gives the scale for the typical energetic separation $\Delta_N^{(d)}$ of states which are directly coupled by the two-body interaction. This has also been found from a study of the two-body random interaction model [56], where only the main structure of the interaction, namely its two-body character and its strength are not taken at random. At U_{c1}, the perturbative description of g_N breaks down, and the level spacing distribution exhibits a crossover from Poisson to the universal Wigner-Dyson behavior. However, because the microscopic Hamiltonian and the corresponding random matrices are very sparse, the correlation of adjacent energy levels does not automatically imply fully ergodic wave-functions. The basis states which can contribute to a given eigenfunction typically lie inside an energy range, given by the spread width $\Gamma_N^{(d)}$ due to the direct coupling. In the intermediate regime, when the spread width of the levels due to the indirect coupling $\Gamma_N^{(i)}$ is between Δ_N and $\Delta_N^{(d)}$, a hierarchical structure appears, characterized by an unequal contribution of the basis states in

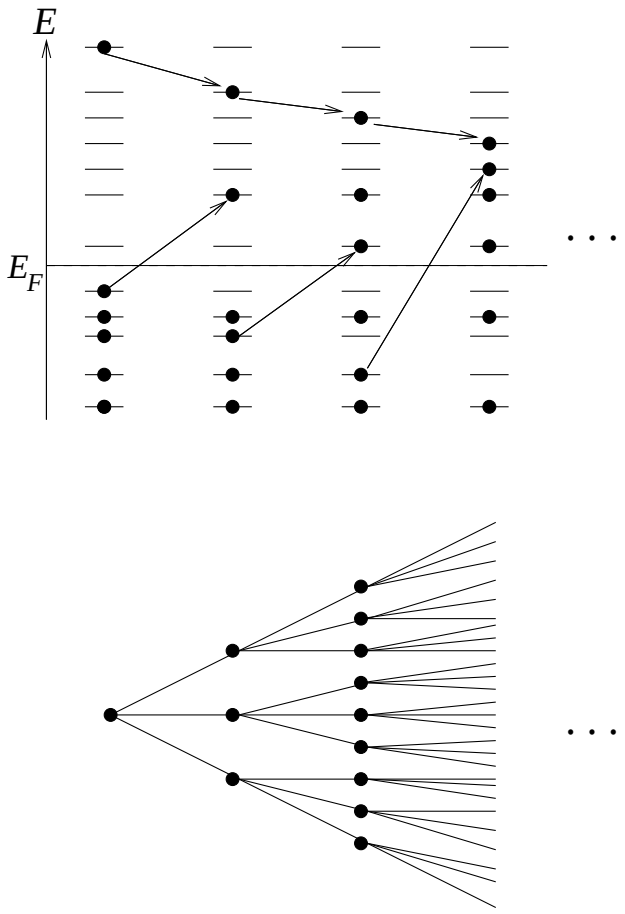

Figure 5: The correspondence of interacting fermions in a finite system (top) and the dynamics on a Cayley tree (bottom).

this range (see Fig. 4). Fully ergodic interacting wave-functions can only be expected [A9] above the second threshold U_{c2}, where the sparseness of the Hamiltonian becomes irrelevant.

3.2 Life-time of quasi-particles

The mixing of the non-interacting states and the correlation of few-body levels is related to the life-time of quasi-particles in a quantum dot [105]. When the non-interacting levels are not mixed, the lifetime of a quasi-particle excitation is infinite, while strong mixing corresponds to fast decay. A mapping [8] of the decay of a quasi-particle excitation in Fock space onto the dynamics of one particle on a disordered Cayley tree was proposed (see Fig. 5). Here, each non-interacting many-body basis state corresponds to one site of

the Cayley tree. The basis states are ordered hierarchically according to the number of particle-hole pairs present. The two-body interaction between the particles couples basis states which differ by only one particle-hole pair and these couplings are translated into finite hopping terms along the branches of the tree.

The well-known localization-delocalization transition on the Cayley tree [1] was then proposed [8] to be relevant for the life-time of quasi-particles and to suppress their Fermi's golden rule decay at low excitation energy. Two different excitation energy scales were found [8] which might correspond to the two interaction scales in the spectral properties of few-particle systems [A9]. The lower one corresponds to the energy below which the life-time of the excitation becomes infinite. Subsequent work [56, 82] indicated that the localization transition is smooth rather than abrupt and the parameter dependence of this energy scale was found to be modified due to the finite number of available states in finite systems.

The mapping of the quasi-particle decay to a Cayley tree is not exact, and direct numerical investigations of a microscopic Hamiltonian [18] and the two-body random interaction model [80], unfortunately limited to small system size, have not succeeded to prove the existence of the transition in the life-time. The application of a different approach [43], keeping some correlations in the many-body energies and the coupling matrix elements [74], indicates however a breakdown of standard Fermi's golden rule decay at low excitation energy.

Since they allow to detect the onset of quantum chaos in many-body systems, studies of the spectral fluctuations in many-body systems are also relevant for possible quantum computers. A quantum computer ideally consists of a large number of independent two-level systems (qubits), and is operated by a controlled switching on and off of the inter-qubit couplings. However, the presence of imperfections and residual interactions between the qubits could make the many-qubit system chaotic, very much like a many-body system of interacting fermions and destroy its operability [101].

4 Correlated fermions in disordered samples

However, the few-particle results presented above are not directly applicable to the electronic transport properties of disordered materials. This is because the many-body properties at finite density do not follow directly from the few-body properties. Furthermore, the size and the geometry of the system cannot be neglected. Another important drawback is the energy of the states under investigation. Almost all of the early work in this field, described in the previous sections, considers two- or few-body states around the center of the many-body band $E = 0$, where the density of states depends only weakly on energy.

This choice was motivated by the fact that the used analytical and numerical techniques (which were developed for studies of disordered one-particle problems) are usually most reliable in the band center. In particular, the quantities related to spectral statistics are more difficult to study in the vicinity of the band edges.

On the other hand, the experimentally observed electronic properties of solid state devices which may be influenced by phase-coherent mesoscopic effects (see section 1.1) are dominated by the properties of the many-body ground state and the lowest-lying excited states. These may have properties which are completely different from those of the states in the many-body band center. In addition, they sometimes cannot be characterized by the usual methods. For example, an investigation of the "chaoticity" of the many-body ground state by comparing the level spacing statistics with universal random matrix theory results is not possible and another measure is needed. Such a quantity will be proposed in section 5. It is an important issue to investigate many-body ground state properties at finite density in order to study whether the interaction effects in disordered systems, like the delocalization of the wave-functions, can be important for the electronic transport in real samples.

A first approach consists in the consideration of a frozen non-interacting filled Fermi-sea, and to study few interacting (quasi-)particles at low excitation energy above the Fermi-sea [52, 53, 119, 8, 57]. However, this is not completely satisfying since the assumption of a non-interacting Fermi-sea and the omission of the interaction between the particles above and in the sea cannot be justified in low-dimensional strongly interacting systems where on the contrary a strongly correlated ground state is expected. This necessitates to study new quantities and to develop new methods for the study of the many-body ground state in disordered systems.

The Lanczos algorithm used for the "exact" numerical diagonalization of the two- and few-particle problem converges much more rapidly at the band edge than in the band center. However, the computer time and the amount

of memory required by the algorithm increases strongly with the number of particles considered such that the method remains nevertheless restricted to rather low numbers of particles.

A modern numerical method which is very well adapted to the study of the ground state of many-body problems is the density matrix renormalization group (DMRG) algorithm [125, 96, 88]. Its main idea is to perform an iterative projection onto a target state (usually the ground state), thereby keeping only the states with the best projection onto the expected state, before the problem is finally diagonalized in this basis of reduced size. Even though this is an approximative method, it can reach an impressive accuracy, close to direct diagonalization, but for much larger systems and particle numbers. On the other hand, the algorithm is intrinsically one-dimensional and offers the best performance for one- or quasi one-dimensional problems.

4.1 One-dimensional systems

The emergence of this new method allows for the precise numerical investigation of different kinds of one-dimensional quantum many-body problems like spin chains, Kondo models, and correlated fermion systems [88]. We focus on the case of correlated spinless fermions in a disordered chain, since this situation is exactly the desired generalization of the two-interacting-particle problem towards the low-energy properties at finite particle density.

A convenient quantity to characterize the transport properties of such a chain is the Drude weight which we present in the following section.

4.1.1 The Drude weight

The localized character of an electron system determines the behavior of the conductance, which is a transport property, as well as the persistent current, which is a thermodynamic property. As first shown by Kohn [64], in the zero temperature limit, both properties can be related. The basic ingredients of such a relationship for the particular case of the insulating regime are presented in the following. The linear response of the electronic system to a spatially uniform, time-dependent electric field is the frequency-dependent conductivity

$$\sigma(\omega) = \sigma_1(\omega) + \mathrm{i}\sigma_2(\omega)\,. \tag{24}$$

Closing a one-dimensional system of L sites to a ring threaded by a magnetic flux Φ and imposing periodic boundary conditions leads to a flux-dependent many-particle ground state energy $E(\Phi)$. We choose units such that $\hbar = e^2 = c = a = 1$ (a is the lattice constant), and $\Phi = 2\pi$ corresponds to one flux quantum threading the ring. Kohn showed that the

second derivative of $E(\Phi)$ (the charge stiffness or Kohn curvature)

$$D_c = L \left(\frac{\mathrm{d}^2 E(\Phi)}{\mathrm{d}\Phi^2} \right) \Bigg|_{\Phi=0} \tag{25}$$

is related to the imaginary part $\sigma_2(\omega)$ of the conductivity through

$$D_c = \lim_{\omega \to 0} \omega \sigma_2(\omega) \,. \tag{26}$$

Using the Kramers-Kronig relations between the real and the imaginary part of the conductivity, it is found that D_c also gives the weight of the zero-frequency peak in the real part of the conductivity

$$\sigma_1(\omega) = \pi D_c \delta(\omega) + \sigma_1^{\mathrm{reg}}(\omega) \,. \tag{27}$$

Therefore, D_c is sometimes called the Drude weight. This establishes a link between transport properties and persistent currents since the latter is given by the flux-dependence of the ground state energy.

In the insulating regime the amplitude of the flux-dependent oscillation of the ground state energy is much smaller than the energy gap between the many-body ground state and the first excited state. Therefore, it is easy to see that a perturbation theory in the hopping matrix elements across the boundary [A11] is enough to describe the flux dependence of the ground state energy, which yields

$$E(\Phi) = E(0) - \frac{\Delta E}{2}(1 - \cos \Phi) \,. \tag{28}$$

Here, $\Delta E = E(0) - E(\pi)$ can also be interpreted as the difference of ground state energies between periodic ($\Phi = 0$) and anti-periodic ($\Phi = \pi$) boundary conditions since a magnetic flux through the ring is equivalent to introducing a change of the boundary conditions. In one dimension, the sign of ΔE depends only on the parity of the number of particles N ($\Delta E < 0$ for odd N and $\Delta E > 0$ for even N) [72].

The simple Φ-dependence of the ground state energy in equation (28) allows to relate ΔE to the persistent current

$$J(\Phi) = -\frac{\mathrm{d}E(\Phi)}{\mathrm{d}\Phi} = \frac{\Delta E}{2} \sin \Phi \,, \tag{29}$$

and the Drude weight

$$D_c = -\frac{L}{2} \Delta E \,. \tag{30}$$

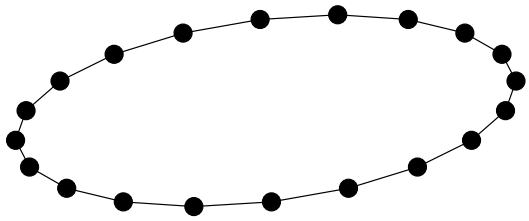

Figure 6: A one-dimensional chain, closed to a ring.

The phase sensitivity

$$D = (-1)^N \frac{L}{2} \Delta E \,, \tag{31}$$

is a derived quantity, that for a one-dimensional ring is simply given by the absolute value of the Drude weight. It is a widely used quantity to characterize the transport properties of many-body systems at zero temperature, and at the same time it determines the magnitude of the persistent current.

The possibility of a negative charge stiffness (or D_c) arising for spinless fermion and Hubbard rings [107, 41] indicates that the orbital response may be paramagnetic. This seems to be a peculiar behavior since D_c is linked to the zero-frequency behavior of σ_1. We however deal here with a system having ring topology, and furthermore no reservoirs and no dissipation are taken into account in the model. As a consequence, it may be questioned whether the computed σ_1 is related to a realistic transport measurement.

4.1.2 Strongly disordered chain

Clearly, a model of spinless fermions in a one-dimensional chain is not expected to be sufficient to explain experimental results on the enhancement of the persistent currents in metallic quasi-one dimensional rings [73, 26] or the two-dimensional electron gas where the metal-insulator transition has been observed. However, the interplay between disorder/localization and interaction on the ground state can be readily studied in this simple model. The interest on disordered one-dimensional interacting models of fermions (with and without spin) can also be assessed from the large variety of analytical [85, 75, 12, 13, 44, 98, 108, 45] and numerical [87, 2, 23, 63, 113, 114, 97, 59, 28] techniques that have been applied to them.

In the absence of interactions, a one-dimensional disordered system is an Anderson insulator, with the electron wave-functions localized on the scale of the one-particle localization length L_1. The dependence of the conductance on the size L of the system is given by an exponential decrease, the characteristic length scale being determined by L_1. If the system is closed into a ring threaded by a magnetic flux, its orbital response (the persistent current) also scales exponentially with L/L_1 [27].

The problem of spinless fermions in a disorder-free chain with nearest-neighbor interactions is exactly solvable [75]. Attractive interactions lead to an inhomogeneous density of the particles (clustering) in the ground state. Repulsive interactions impose a more homogeneous density (charge density wave). In particular, at half filling strong repulsion imposes a Mott insulator, one gets a finite gap between the ground state energy and the excited levels, and a charge density wave concentrated on alternating sites of the lattice. Disorder tends to distort those arrangements by favoring the occupancy of the low potential sites. The competition between disorder and interactions acquires then a non-trivial character. In the thermodynamic limit (infinite size at constant mean particle density) any amount of disorder destroys the Mott insulator and one obtains large islands of alternating site occupancy separated by defects where the interaction energy has to be paid [98, 87].

Once a one-dimensional interacting system is closed to a ring, its orbital response depends on whether or not the resulting system has rotational invariance. In rotational invariant continuum systems the persistent current does not depend on interactions [85, 108] of any range. But when the rotational invariance is broken by a lattice or by the presence of disorder, interactions can modify the value of the persistent current. The most interesting regime is thus the intermediate one between the Anderson and Mott insulators where all, disorder, interaction and kinetic energy are relevant.

Early numerical work on interacting disordered rings has necessarily been restricted to small samples [85, 84, 23] or it has relayed on strong approximations (like Hartree-Fock [63, 59]). Direct diagonalization of small systems (lattices with $L = 6$ and 10 sites at half filling) with Coulomb interaction [2] yielded, when averaged over impurity realizations, a persistent current that is suppressed by effects of the interactions. Only at strong disorder and weak interaction strength, a weak enhancement was found. Direct diagonalization in one-dimensional rings of spinless fermions with short-range interactions in lattices of up to 20 sites [23] found that both, disorder and interactions, always decrease the persistent current by localizing the electrons. This simulation deals with values of the disorder that are not strong in comparison with the typical kinetic energy of the electrons. Similar results have been obtained in this regime using the density matrix renormalization group

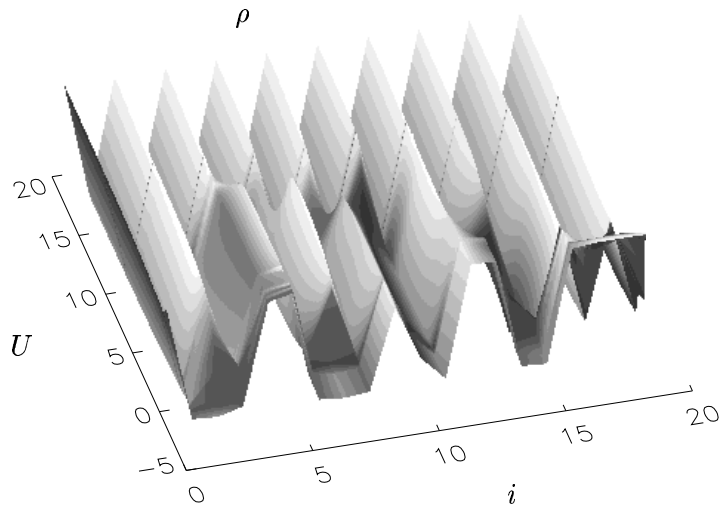

Figure 7: The particle density ρ as a function of the position i and the interaction strength U for 10 spinless fermions on a strongly disordered chain consisting of 20 sites (one configuration drawn with $W = 9$). The density changes in abrupt steps from the inhomogeneous distribution adapted to the disordered potential at small U (in the Anderson insulator regime) to the regular array characteristic of the Mott insulator at large U. The figure is taken from [A10].

(DMRG) method [97]. Hartree-Fock calculations also yielded a suppression of the persistent current as the strength of the interaction increases [63].

Let us consider now spinless fermions on a chain with nearest-neighbor interaction

$$H = -t \sum_{i=1}^{L} (c_i^+ c_{i-1} + c_{i-1}^+ c_i) + \sum_{i=1}^{L} v_i n_i + U \sum_{i=1}^{L} n_i n_{i-1} \tag{32}$$

and twisted boundary conditions, $c_0 = \exp(\mathrm{i}\Phi) c_L$. The on-site random energies v_i are again drawn from a box distribution of width W. The strength of the disorder W and the interaction U are measured in units of the kinetic energy scale ($t = 1$). The use of twisted boundary conditions allows to represent a ring of L sites pierced by a flux Φ.

At large disorder, the numerical computations of the phase sensitivity using DMRG yield other results than the previous numerical studies performed at weaker disorder, and thereby indicate the existence of a new and different regime. When increasing the strength of the interactions, one finds

abrupt charge reorganizations (see Figure 7) of the many-body ground state (at sample-dependent values of the interaction strength) which are associated with anomalously large persistent currents [A10, A11] appearing in individual samples. It turns out that in order to obtain the strong enhancement effect, the disorder must be strong enough to have a one-particle localization on the length scale of the typical distance between particles. In the ensemble average, a small enhancement of the phase sensitivity is obtained at an interaction strength of the order of the kinetic energy scale [A10] and an investigation of the density-density correlation function [A11] shows that the many-body ground state is closest to a liquid state at the same interaction strength. This enhancement persists in the thermodynamic limit taken at constant particle density and can be expressed as an enhancement of the many-body localization length due to the interactions [A11, A12]. Recent analytical work [A11] allows to understand several aspects of these numerical findings.

An attempt [60] to obtain the same results within Hartree-Fock calculations shows good agreement with the exact data from the DMRG calculations at weak interaction strength, but fails at stronger interaction when the ground state becomes strongly correlated.

A sizeable increase of the persistent current with the strength of the interaction had been obtained for moderately disordered one-dimensional electronic systems with spin (Anderson-Hubbard model) from renormalization group approaches [44] or perturbation theoretical and numerical calculations [28]. However the spin does not seem to be a necessary ingredient for obtaining an enhancement of persistent currents due to interactions. Even without spin, repulsive interactions can increase the persistent current, provided the disorder is important enough [A10, A11]. This allows to expect an even more dramatic increase at strong disorder in models with spin.

It is interesting to remark that the enhancement of transport properties by the effect of a repulsive interaction has been proposed in other contexts than this one. A system with strong binary disorder where the two possible values of the disorder give rise to two separated bands presents in the absence of interactions a gap if the filling is such that the lower band is filled completely [115]. The broadening of the bands due to the interaction can then lead to an overlap allowing for metallic behavior.

4.2 Two-dimensional systems

The conclusions of the work on fermions in one-dimensional disordered chains suggest an increase of the localization length at moderate interaction strength, between the regimes of the Anderson insulator and the Mott

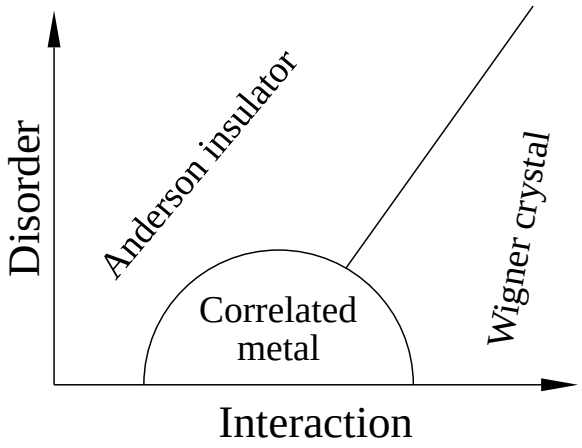

Figure 8: Schematic phase diagram proposed for disordered interacting two-dimensional systems.

insulator. However, the system always remains an insulator. According to the scaling theory of localization (see section 1.2.1), two dimensions are the limiting case for obtaining non-interacting systems which are localized at arbitrary disorder. A delocalization due to electron-electron interaction may therefore be expected to have drastic effects in two dimensions and might lead to the appearance of a metallic phase between the non-interacting Anderson insulator and the strong-interaction limit, the pinned Wigner crystal. Such a scenario, sketched in figure 8 is supported by numerical calculations using exact diagonalization of small two-dimensional clusters with Coulomb-interacting fermions [14], and diagonalizing in a Hartree-Fock basis [116] in somewhat larger systems. The metallic nature of the intermediate phase is confirmed by finite-size scaling calculations [122] and level statistics for the lowest excitations [16]. Since the effective interaction strength (and also the effective disorder strength) in a realistic system increase when the charge carrier density is decreased, this suggests that at low density the experiments might be in the pinned Wigner crystal regime and pass through a metallic phase before becoming an Anderson insulator at high carrier densities. This scenario is supported by some of the experiments [50].

The exact diagonalization calculations of the ground state for a few particles in small two-dimensional clusters [14] show charge reorganizations accompanied by strongly enhanced (persistent) currents, exactly as it was observed in one dimensional rings [A10]. A recent experiment [51] shows strong fluctuations of the local compressibility in the insulating regime at low carrier density. These fluctuations might be a direct consequence of the charge

reorganizations predicted by the calculations.

4.2.1 Persistent currents at strong Coulomb interaction

Since the insulating side at low charge carrier density of the metal-insulator transition in two-dimensional samples might be a pinned Wigner crystal, it is interesting to consider this limit in more detail. Using a disordered two-dimensional lattice model closed to a torus topology with Coulomb interaction between the particles, it is found that the persistent current decreases strongly when the interaction strength between the particles is enhanced [14, 17, 19]. This is a signature of the Wigner crystal becoming more and more rigid and the pinning by the disorder more efficient.

At intermediate interaction strength, when the Wigner crystal is formed, a change in the distribution of the local currents is observed [14, 19]. While the local currents have more or less random orientation at weak interaction, their direction becomes well-defined and restricted to the longitudinal direction in the Wigner crystal regime, independent of the disorder realization. The transverse currents decreasing with the interaction strength faster than the longitudinal ones, this alignment of the local currents has been proposed [15] as a signature of the metal-insulator transition in two dimensions.

In a theoretical study of the persistent current in the strongly interacting Wigner crystal regime on two-dimensional lattices [114, A13], a perturbative treatment of the hopping matrix elements allows to obtain the leading contributions to the persistent current. The decrease of the persistent current is found to be given by power laws, the length of the system defining the disorder-independent exponent. Furthermore, the sign of the persistent current becomes well-defined in the Wigner crystal limit and follows only from the geometry of the Wigner crystal many-body ground state [A13]. Simple rules, similar to the ones which are generally valid in one dimension [72] can be found for this sign.

However, further research is necessary to clarify the nature and the origin of the metallic phase in two-dimensional systems.

5 Phase space analysis of wave-functions

As shown in the previous sections, electron-electron interactions induce correlations in the level statistics of many-body systems. If the one-particle eigenstates are localized due to strong disorder, the spectral correlations can be accompanied by a delocalization of the particles in real space and a more liquid structure of the many-body charge density. Similar spectral correlations are observed for the quantum energy levels of classically chaotic systems [48].

For classical chaos, on the other hand, it is convenient to consider the classical dynamics in phase space $(\vec{x}, \vec{p})$. This suggests that useful informations could be drawn from the phase space structure of quantum wave-functions. However, the real space representation $\psi(\vec{x})$ as well as the momentum space representation $\tilde{\psi}(\vec{k} = \vec{p}/\hbar)$ contain each the full information about the quantum state. Furthermore, according to Heisenbergs uncertainty relation, position and momentum of a quantum particle cannot be well-defined simultaneously.

5.1 Quantum mechanical phase space concepts

It is nevertheless possible to construct quantities which depend on both, position and momentum $p = \hbar k$, allowing to study the structure of the wave functions in phase space (x, k). The Husimi density characterizing a given state $|\psi\rangle$,

$$\rho_{\mathrm{H}}(x_0, k_0) = \left|\langle x_0, k_0|\psi\rangle\right|^2 , \tag{33}$$

is given by the projection of $|\psi\rangle$ on minimal uncertainty states $|x_0, k_0\rangle$. In [A14], Gaussian wave packets with variance σ^2 are used, centered around position x_0 and wave number k_0. In real space representation, these states are given by

$$\langle x|x_0, k_0\rangle = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} \exp\left(-\frac{(x - x_0)^2}{4\sigma^2} + \mathrm{i}k_0 x\right) . \tag{34}$$

This definition of ρ_{H} yields the normalization $\int \frac{\mathrm{d}x\mathrm{d}k}{2\pi} \rho_{\mathrm{H}}(x, k) = 1$. Thus, the Husimi-distribution is obtained by projecting the wave-function onto states which are localized in phase space, with minimal uncertainty, thereby guaranteeing the best possible resolution. This Husimi-distribution can be used to visualize the phase space structure of quantum states. Its analysis allows to detect structures in the wave functions which are related to the underlying classical dynamics in phase space and can be used to determine the chaoticity of quantum systems [109]. Since ρ_{H} is always non-negative, the Wehrl

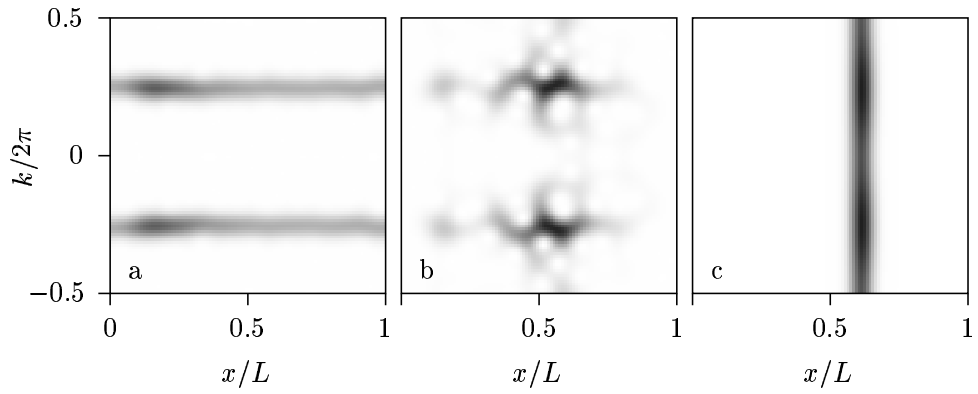

Figure 9: Husimi-distribution as a function of position x and momentum k for one-particle eigenfunctions in a one-dimensional Anderson-model containing $L = 100$ sites. At weak disorder $W = 0.5$ (a) the distribution is localized in momentum but completely delocalized in real space. At strong disorder $W = 300$ (c), the state is localized in real space but localized in momentum space. At intermediate disorder $W = 3$ (b), the state is more or less delocalized in real and momentum space (The figure is taken from [A14]).

entropy

$$S_{\mathrm{H}} = -\int \frac{\mathrm{d}x\mathrm{d}k}{2\pi}\, \rho_{\mathrm{H}}(x,k) \ln\left[\rho_{\mathrm{H}}(x,k)\right] \tag{35}$$

can be defined [124, 81]. S_{H} is a measure of the phase space volume occupied by the quantum state. It has been shown for the driven rotor that the Wehrl entropy of individual states is connected to the energy level statistics. Both quantities can be used to distinguish between the chaotic and the integrable regime [46]. A very similar system, the kicked rotor, can be mapped onto the Anderson model [37], suggesting that the Wehrl entropy is a useful quantity for the characterization of the eigenstates of the Anderson model.

5.2 Disordered systems in phase space

Examples of the Husimi densities calculated [A14] for eigenfunctions of one-dimensional Anderson models with different disorder strength are shown in Fig. 9. One clearly observes the simple phase space structure corresponding to plane waves with fixed momentum in the ballistic limit of low disorder. Similarly, at strong disorder, strong real space localization fixes the position and leads to a simple Husimi density. While these two limits correspond to regular classical dynamics, the case of intermediate disorder strength is less obvious and the Husimi density, covering a larger part of the phase space,

has a complicated structure.

States corresponding to regular behavior are expected to be mainly concentrated on a fraction of the phase space while chaotic states are expected to be distributed over larger parts of the phase space. A measure for the phase space occupation of a state is the Wehrl entropy [124]. This entropy can be used to characterize the chaotic character of individual quantum states. In contrast to level statistics, it may be used to describe the many-body ground state of an interacting system.

That this kind of analysis yields useful results for disordered systems has been shown [A14] by applying it to the eigenstates of the one- and two-dimensional Anderson model. In one dimension, the ensemble averaged phase space entropy shows a monotonous cross-over from the ballistic to the localized regime. In two dimensions, the diffusive regime is known to appear between the ballistic and the localized one. The average phase space entropy assumes a higher value there, which allows to detect the diffusive chaotic regime and to distinguish it from the regular ballistic and localized regimes.

It will be interesting to use this method for the investigation of systems exhibiting critical statistics, and to shed light on the many-body correlations induced by electron-electron interactions in disordered systems.

6 Summary and outlook

Mesoscopic systems with disorder and interaction show a very rich variety of phenomena in different parameter regimes due to the competition of the kinetic energy scale with the disorder potential energy and the interaction energy. To treat the regime where all these energy scales are important is a challenge for theoretical physics. The approach presented here starts from the case of very few particles, and allows for the numerical treatment of some properties like the level statistics and the extension of the few-body wavefunctions. This allows to test ideas and analytical model calculations which can then be extended to larger particle numbers.

We have reviewed extensive numerical investigations and the development of random matrix models for the delocalization of the many-body wavefunctions and the correlations induced in the many-body spectrum due to repulsive interactions of intermediate strength. A study of the persistent current of many correlated particles in strongly disordered one-dimensional rings showed that the delocalization due to interactions is a relevant mechanism not only for few particles, but also for the ground state of many particles. The delocalization effect persists in the thermodynamic limit and might be a step towards understanding the nature of the experimentally observed metallic phase in two dimensions.

However, more work and a deeper understanding of the electronic correlations induced by the electron-electron interactions will be necessary in order to quantitatively understand the experimental features. In particular, it seems to be important to include the spin of the electrons and to take into account the full long-range Coulomb interaction in two-dimensional models. Since this will require to study *a priori* more complicated models, the knowledge of the already complex behavior of basic few-body systems with disorder and interaction presented here will be an important guide for future investigations.

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Appendix: Selected Publications

[A1] D. Weinmann, A. Müller-Groeling, J.-L. Pichard, and K. Frahm, *$h/2e$-Oscillations for Correlated Electron Pairs in Disordered Mesoscopic Rings*, Phys. Rev. Lett. **75**, 1598–1601 (1995).

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[A4] D. Weinmann and J.-L. Pichard, *Level Statistics and Localization for Two Interacting Particles in a Random Potential*, Phys. Rev. Lett. **77**, 1556–1559 (1996).

[A5] K. Frahm, A. Müller-Groeling, J.-L. Pichard, and D. Weinmann, *Quantum Transport with Disorder and Interaction: The Two Particle Case*, in *Correlated Fermions and Transport in Mesoscopic Systems*, ed. by T. Martin, G. Montambaux and J. Trân Than Vân, Editions Frontières (Gif-sur-Yvette 1996), pages 221–234.

[A6] X. Waintal, D. Weinmann, and J.-L. Pichard, *Two interacting particles in a disordered chain II: Critical statistics and maximum mixing of the one body states*, Eur. Phys. J. B **7**, 451–456 (1999).

[A7] A. Wobst and D. Weinmann, *Two interacting particles in a disordered chain IV: Scaling of level curvatures*, Eur. Phys. J. B **10**, 159–167 (1999).

[A8] J.-L. Pichard, D. Weinmann, S. De Toro Arias, R. A. Jalabert, P. Schmitteckert, X. Waintal, and A. Wobst, *Interplay disorder-interaction in one dimensional quantum models*, Ann. Phys. (Leipzig) **7**, 462–482 (1998).

[A9] D. Weinmann, J.-L. Pichard, and Y. Imry, *Thouless numbers for few-particle systems with disorder and interactions*, J. Phys. I France **7**, 1559-1581 (1997).

[A10] P. Schmitteckert, R. A. Jalabert, D. Weinmann, and J.-L. Pichard, *From the Fermi glass towards the Mott insulator in one dimension:*

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[A12] R. A. Jalabert, D. Weinmann, and J.-L. Pichard, *Partial delocalization of the ground state by repulsive interactions in a disordered chain*, to appear in Physica E (2001).

[A13] F. Selva and D. Weinmann, *Persistent currents for Coulomb interacting electrons on 2d disordered lattices: Sign and interaction dependence in the Wigner crystal regime*, Eur. Phys. J. B **18**, 137–148 (2000).

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Erklärung

Ich versichere hiermit, dass ich selbst einen erheblichen Anteil der wissenschaftlichen Leistung zu den im Anhang eingebundenen Veröffentlichungen erbracht habe.

Augsburg, den 4. Dezember 2000