

## Formulaire

### Systèmes de coordonnées

#### Coordonnées cartésiennes

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}; \quad d\tau = dx dy dz$$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}} \\ \nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\ \nabla \times \mathbf{v} &= \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}} \\ \nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \end{aligned}$$

#### Coordonnées sphériques

$$\begin{aligned} x &= r \sin \theta \cos \varphi & \hat{\mathbf{x}} &= \sin \theta \cos \varphi \hat{\mathbf{r}} + \cos \theta \cos \varphi \hat{\boldsymbol{\theta}} - \sin \varphi \hat{\boldsymbol{\phi}} \\ y &= r \sin \theta \sin \varphi & \hat{\mathbf{y}} &= \sin \theta \sin \varphi \hat{\mathbf{r}} + \cos \theta \sin \varphi \hat{\boldsymbol{\theta}} + \cos \varphi \hat{\boldsymbol{\phi}} \\ z &= r \cos \theta & \hat{\mathbf{z}} &= \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} & \hat{\mathbf{r}} &= \sin \theta \cos \varphi \hat{\mathbf{x}} + \sin \theta \sin \varphi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ \theta &= \arctan(\sqrt{x^2 + y^2}/z) & \hat{\boldsymbol{\theta}} &= \cos \theta \cos \varphi \hat{\mathbf{x}} + \cos \theta \sin \varphi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}} \\ \varphi &= \arctan(y/x) & \hat{\boldsymbol{\phi}} &= -\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}} \end{aligned}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\varphi \hat{\boldsymbol{\phi}}; \quad d\tau = r^2 \sin \theta dr d\theta d\varphi$$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \hat{\boldsymbol{\phi}} \\ \nabla \cdot \mathbf{v} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\varphi}{\partial \varphi} \\ \nabla \times \mathbf{v} &= \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta v_\varphi) - \frac{\partial v_\theta}{\partial \varphi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r} (r v_\varphi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}} \\ \nabla^2 f &= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2} \end{aligned}$$

#### Coordonnées cylindriques

$$\begin{aligned} x &= r \sin \theta & \hat{\mathbf{x}} &= \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\boldsymbol{\theta}} \\ y &= r \cos \theta & \hat{\mathbf{y}} &= \sin \theta \hat{\mathbf{r}} + \cos \theta \hat{\boldsymbol{\theta}} \\ z &= z & \hat{\mathbf{z}} &= \hat{\mathbf{z}} \end{aligned}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan(y/x)$$

$$z = z$$

$$\hat{\mathbf{r}} = \cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\theta}} = -\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{y}}$$

$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + dz \hat{\mathbf{z}}; \quad d\tau = r dr d\theta dz$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \mathbf{v} = \left[ \frac{1}{r} \frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \right] \hat{\mathbf{r}} + \left[ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (rv_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

## Identités vectorielles

### Triples produits

- (1)  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$
- (2)  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

### Règles du produit

- (3)  $\nabla(fg) = f(\nabla g) + g(\nabla f)$
- (4)  $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A}$
- (5)  $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$
- (6)  $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$
- (7)  $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$
- (8)  $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

### Dérivées secondes

- (9)  $\nabla \cdot (\nabla \times \mathbf{A}) = 0$
- (10)  $\nabla \times (\nabla f) = 0$
- (11)  $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

## Théorèmes fondamentaux

**Théorème du gradient :**  $\int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{l} \cdot \nabla f = f(\mathbf{b}) - f(\mathbf{a})$

**Théorème de la divergence :**  $\int_{\mathcal{V}} d\tau (\nabla \cdot \mathbf{A}) = \oint_{\mathcal{S}} d\mathbf{a} \cdot \mathbf{A}$  (Green-Ostrogradski)

**Théorème du rotationnel :**  $\int_{\mathcal{S}} d\mathbf{a} \cdot (\nabla \times \mathbf{A}) = \oint_{\mathcal{P}} d\mathbf{l} \cdot \mathbf{A}$  (Stokes)

# Équations fondamentales de l'électrodynamique

## Équations de Maxwell

Forme générale :

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{1}{\epsilon_0} \rho \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Dans la matière :

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho_f \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

## Champs auxiliaires

Définitions :

$$\begin{aligned}\mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} &= \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}\end{aligned}$$

Milieux linéaires :

$$\begin{aligned}\mathbf{P} &= \epsilon_0 \chi_e \mathbf{E}, & \mathbf{D} &= \epsilon \mathbf{E} \\ \mathbf{M} &= \chi_m \mathbf{H}, & \mathbf{H} &= \frac{1}{\mu} \mathbf{B}\end{aligned}$$

## Potentiels

$$\begin{aligned}\mathbf{E} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A}\end{aligned}$$

## Force de Lorentz

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

## Constantes fondamentales

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$	(permittivité du vide)
$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$	(perméabilité du vide)
$c = 3.00 \times 10^8 \text{ m/s}$	(vitesse de la lumière)
$e = 1.60 \times 10^{-19} \text{ C}$	(charge de l'électron)
$m = 9.11 \times 10^{-31} \text{ kg}$	(masse de l'électron)